METHOD FOR DETERMINING THE FOCAL LENGTH IN A DIGITAL NON-METRIC CAMERA

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Summary
The authors analysed the methods for determining the focal length in digital cameras (DC) and proposed a method, according to which the control-measuring grid (CMG) is located vertically at a distance from the DC and a photograph of the CMG is made, then the latter (the CMG) is moved along the optical axis of the digital camera to the distance, which is fixed with a micrometre screw and, repeatedly, a series of photographs of the CMG is taken. After that, on the received digital images, the coordinates are measured on the corresponding intersections of the CMG, and subsequently, the focal length for the DC is determined. The a priori estimation of accuracy of determining the focal length for the DC by the proposed method is calculated. For approbation of the method, the focal length in the following cameras was determined: Canon EOS 350D, Canon EOS 450D, Canon EOS 5D.

Keywords
focal length • digital non-metric camera

1. Introduction
The main objective is to develop and investigate the method for determining the focal length in the digital camera.

An analysis of recent research and publications devoted to solving this problem
Today, the use of the non-metric digital cameras is still relevant and continues to be explored. In contradistinction to the expensive metric cameras, non-metric ones need to be calibrated. Calibration of the DC is implemented with the aim of using it as a measuring instrument for aerial surveys, conducted from an unmanned aerial vehicle (UAV) and for further metric processing of images taken by this camera.

The issue of research and analysis of methods for determining the focal length of the DC is discussed in a number of papers. We will review and analyse some of them.

Several articles [Levyts’kyy 2008, Levyts’kyy and Sobolevs’kyy 2008], present the main stages of calibration as follows:
1. Selecting and creating a testing ground (test object) for taking photos. The test object can be plane or volumetric, which determines the subsequent technique of images processing.

2. Measurement of the testing ground and processing of the results.

3. Taking photos of the testing ground and obtaining at least 12–15 images, taken from different angles for more accurate determination of the parameters.

4. Images processing, using the software based on the solution of the tasks, which can be solved either by means of linear or non-linear methods.

5. Definition of calibration parameters. For non-metric digital cameras, self-calibration parameters include focal length, principal point coordinates, and systematic errors that cause shifting of image points in radial and tangential directions (distortion of the lens).

6. Analysis of the results and saving them in the appropriate format for further use.

The proposed method of calibration of digital non-metric cameras makes it possible to facilitate image processing and increase the reliability of obtaining the undistorted image of the object being investigated. However, it should be noted that implementation of this technology does not take into account the effect from correlation between elements of external orientation and the elements of internal orientation, which will definitely cause deterioration of the accuracy of the desired elements.

The paper [Patent na vynakhid No. 99984, Glotov and Pashchetnyk 2012] provides a method for determining the focal length of the DC, according to which CMG is established from the lens of DC at fixed distances that are not located at infinity from this lens. Then by coordinating the values of horizontal and vertical angles, the equivalent focal distance is determined, by which conclusion is made about the real focal length of the DC. However, in this procedure, a total station is used, which greatly reduces the cost, efficiency and feasibility of the method.

The calibration method [Zhang 2000, Zhang, Tsai 1987] requires that a planar pattern be placed in at least 2 different orientations in front of the DC. You can move a camera or a planar pattern. The motion does not have to be known. The distortion of the lens is modelled. After that, the elements of external orientation (EEO), elements of internal orientation (EIO), and distortion parameters are determined. Compared to classical methods, this method does not use expensive equipment, but since the camera calibration occurs using a planar pattern, it should be noted that calibration will only be possible under laboratory conditions.

In the paper [Patent na vynakhid No. 107756 Glotov and Pashchetnyk 2015], the focal lengths of the DC are determined as follows: CMG is set horizontally, filming is performed, then a line segment is constructed graphically, with dimensions coinciding with the focal length of the DC, on the resulting image of the CMG. However, in this method, the CMG is set horizontally, which makes it difficult to focus on the entire area of the photograph's subject, since the coordinates of the points of the CMG are at different distances relative to the main optical axis of the DC. This fact in turn reduces
the accuracy of measurements on the received digital images, which leads to lowering the accuracy of determining the focal length of the camera being investigated.

One of the promising methods of calibration of images, and one which is available in our conditions, is a method based on determining the characteristics of the central projection of images and its distortion by images of a special stand or control area of the locality, where the coordinates X, Y, Z of a large number of points are determined. In paper [Sholomickij and Shatohin 2002], a place for a calibrated stand was chosen on the wall of one of the classrooms. Dimensions of the stand were 3200 × 4800 mm (9 × 13 marks). On the wall, with a step equal to 40 cm in the nodes of the grid, metal marks were fixed, 45 × 45 mm in size. To solve the problem of calibration of images, one needs to know the spatial coordinates X, Y, Z of all points of the polygon. Consequently, the first step in solving the task is to carry out measurements of the test stand in order to find the coordinates of all marks in the conventional system. However, in this method, the creation of the stand itself complicated the way of calibrating the camera, therefore in real conditions of aerial survey the calibration will not be possible.

In the publication [Glotov and Pashchetnyk 2013, Patent na vynakhid No. 94376, Glotov and Pashchetnyk 2011], a method is proposed according to which the photography of the CMG is performed; additionally, a mirror is installed on the pivot device, with the ability to deviate the image of the CMG into the camera lens. Subsequently, at the digital photogrammetric station (DPS), “Delta-2” measurements of the corresponding coordinates of the intersection of the CMG on a digital image are carried out, and the horizontal and vertical angles are measured between the directions on the central and other intersections. According to the obtained data, the focal length is calculated. This makes it possible to increase the adaptability, efficiency and accuracy, as well as reduce the cost of determining the focal length. However, the method cannot be applied in the short-wave photogrammetry, since it involves the removal of points of objects located at infinity for the DC lens.

The work [Miheeva 2011] considers the parameters of calibration of CPC lenses and the requirements that must be observed for obtaining quality material. The author notes that the passport of digital cameras does not specify the range of focal lengths, but only the range of segments within which the CCD matrix can move to obtain a sharp image. The paper also provides formulas for determining the length of such segments and focal lengths. However, in order to calibrate the camera, a more hyper focal distance is needed, except when the pictures are taken at close distances.

In papers [Patent na vynakhid No. 23088001, Maljavskij et al. 2007] photogrammetric calibration of the DC for the images of the test object is discussed. The method is based on photographing a test object with two cameras from one point of space. In this case, the EIO of one camera should be known, and the distortion parameters can be neglected, because they are very small. Photos taken by the camera with a known EIO are considered to be the reference. Determination of EIO and distortion parameters of the second camera is performed when comparing the received image with the reference. The disadvantage of the method is the complexity of the calibration, due to the need of having a reference camera with a known EIO and with negligible distortion parameters.
The publication [Patent na vynakhid No. 50155, Glotov and Pashchetnyk 2010] considers the method when a mark with a central cross-section is established with the possibility of its rotation in a vertical plane around the central cross-section, then images of the mark are taken in at least four positions, rotating it by 360°, and the planned EIO is calculated for the coordinates of these intersections for each mark position. However, this method defines only the planned EIO, which limits the information about the DC and does not allow determining of the planned elements and focal lengths simultaneously, which is especially important for aerial surveying by an unmanned aerial vehicle (UAV); therefore, that reduces the accuracy of the calculation of the spatial coordinates of the points of research objects.

Having reviewed the literary sources, it is convincingly clear that, firstly, the chosen subject of research is relevant today, which, moreover, is confirmed by a large number of publications devoted to this topic. Secondly, there is a need for further development of ways to define the elements of internal orientation, since there is an urgent need to increase productivity and reduce the cost of the methods. This is due to the fact that, in the context of using non-metric digital cameras, companies today lack the appropriate nomenclature to research photographic equipment.

2. Presentation of the main materials

The method proposed by the authors can be implemented for determining the focal length of non-metric digital imaging systems. Subsequently, it can be used in various fields of science and technology, for example, to calibrate the DCs used for topographical aerial surveying with UAVs, which will enable to improve the accuracy of coordinates determination of the object on the territory.

The method for determining the focal length is as follows. We place DC on the stand on a tripod and bring it to the working condition – we perform levelling using the overhead level. On a cart, CMG is arranged vertically at a certain distance, so that the main optical axis is approximately perpendicular to the centre of the CMG; then, a photograph is taken. After that, CMG is moved to a distance fixed by the measuring device, and another photograph is taken.

Figure 1 schematically depicts the method for determining the focal length of the DC.

According to the coordinates of a, b, a’, b’, which are measured on the received digital images, we compute the lengths of the lines on the CMG between intersections. It is assumed that a'b' = l', ab = l, A'B' = CD, KP = d. Then the value of the focal length of the DC is determined as follows.

Since triangles a'b'S and A'B'S are similar, and triangle abS is similar to the triangle CDS, then the coefficients of the similarity $K_1, K_2$ are determined:

$$K_1 = \frac{L}{l'}$$

$$K_2 = \frac{L}{l}$$

(1)

(2)
From the triangle $SOa$, we calculate $SO$:

$$SO = aO \cot \alpha = 0.5 \ l \ cot \alpha.$$ 

From the $SPC$ we calculate the cathetus $SP$:

$$SP = CP \ cot \alpha = 0.5 \ L \ cot \alpha,$$

so

$$\frac{SO}{SP} = \frac{l}{L} \quad \text{and} \quad SP = \frac{L}{l} SO = K_2 SO.$$ 

Similarly, from $SOa'$ triangle we calculate $SO$:

$$SO = a'O \ cot \beta = 0.5 \ l' \ cot \beta$$

and from $SKA'$ we calculate the cathetus $SK$:

$$SK = A'K \ cot \beta = 0.5 \ L \ cot \beta.$$ 

Then

$$\frac{SO}{SK} = \frac{l'}{L} \quad \text{and} \quad SK = \frac{L}{l'} SO = K_1 SO.$$ 

As follows

$$SP - SK = K_2 SO = K_1 SO = (K_2 - K_1) SO,$$
on the other hand
\[ SP - SK = KP = d, \]
therefore
\[ (K_2 - K_1)SO = d \quad (3) \]

From formula (3) we determine the focal length \( f = SO \):
\[ f = \frac{d}{K_2 - K_1} \quad (4) \]

According to formulas (1), (2) and (4) we obtain the following:
\[ f = f = \frac{d}{L\left(\frac{1}{l} - \frac{1}{l'}\right)} \quad (5) \]

The technological implementation of the method for determining the focal length of the DC is presented in Figure 2.

Fig. 2. Schematic arrangement of devices and elements of the photograph, where: 1 – micrometer, 2 – CMG, 3 – carriage, 4 – main optical axis, 5 – DC.

The a priori estimation of accuracy of the definition of focal length is calculated using the formula (6), which was obtained from the formula (5):

\[ m_f = \sqrt{\left(\frac{ll'}{L(l' - l)}\right)^2 m_d^2 + \left(\frac{ll'd}{L^2(l' - l)}\right)^2 m_L^2 + \left(\frac{d}{L}\left(\frac{l'}{l' - l}\right)\right)^2 m_r^2 + \left(\frac{d}{L}\left(\frac{l}{l' - l}\right)\right)^2 m_r^2} \quad (6) \]

where:
- \( m_d \) – mean square error (MSE) of the micrometer (±0.005 mm);
- \( m_L \) – mean square error of drawing CMG (±0.005 mm);
- \( m_r \) – MSE of measuring coordinates on the image (±0.002 mm).
In order to test the method, focal lengths were determined for cameras: Canon EOS 350D, Canon EOS 450D and Canon EOS 5D. The focal length of the DC determined by the declared method and the a priori estimate of the accuracy of determining the focal length are given in Table 1.

### Table 1. Accuracy of determining of focal length

<table>
<thead>
<tr>
<th>Camera</th>
<th>(l) [mm]</th>
<th>(l') [mm]</th>
<th>(d) [mm]</th>
<th>(f) [mm]</th>
<th>(m_f) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canon EOS 350D</td>
<td>13.24</td>
<td>17.83</td>
<td>50</td>
<td>51.51</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>8.46</td>
<td>11.17</td>
<td>50</td>
<td>34.94</td>
<td>0.01</td>
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<tr>
<td></td>
<td>6.68</td>
<td>8.83</td>
<td>50</td>
<td>27.41</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>5.17</td>
<td>6.45</td>
<td>35</td>
<td>18.30</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>6.68</td>
<td>8.83</td>
<td>50</td>
<td>27.41</td>
<td>0.02</td>
</tr>
<tr>
<td>Canon EOS 450D</td>
<td>12.22</td>
<td>15.91</td>
<td>50</td>
<td>52.64</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>7.50</td>
<td>9.64</td>
<td>50</td>
<td>33.64</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>6.07</td>
<td>7.81</td>
<td>50</td>
<td>27.33</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>4.14</td>
<td>5.38</td>
<td>50</td>
<td>17.99</td>
<td>0.02</td>
</tr>
<tr>
<td>Canon EOS 5D</td>
<td>6.09</td>
<td>8.16</td>
<td>50</td>
<td>23.92</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Source: authors’ study

Concerning the accuracy of the proposed method in determining the focal length of the DC, investigation was conducted taking into account all the devices and parts that make up the measuring complex. As a result, the following parameters were identified, the disregard of which leads to a significant reduction in accuracy:

1. Non-perpendicularity and deviation of the main optical axis of DC relatively to the CMG;
2. Installation of CMG and CCD matrix of DC in vertical position;
3. Accuracy in determining the coordinates of the intersection of the CMG in the image;
4. Accuracy of the grid calibration;
5. Micrometer accuracy.

For calculating the total accuracy, parameters of errors that affect accuracy will be as follows.

Permissible angle of the non-perpendicularity of the main optical axis relative to the CMG is calculated as

\[
\text{tg}\delta = \frac{r}{L},
\]

where:

- \(r\) – minimum cross-section radius;
- \(L\) – distance from the intersection of the mark of DC to the plane of the CMG

\[
\delta = 8''.
\]
The allowable deviation of the installation of CMG and CCD-matrix of DC in the vertical position is detected (Figure 3).

![Figure 3](source: authors' study)

**Fig. 3.** Deviation of the installation of the CMG and CCD-matrix of the DC in the vertical position, where 1 - CMG, set up in the vertical position, 2 - deviation of the installation of the CMG, 3 - CCD-matrix, set up to the vertical position, 4 - deviation of the installation of the CCD-matrix, 5 - the main optical axis

Size of CMG $L = 50$ mm, permissible deviation of measurement of CMG $\Delta L = 2 \mu$m, thus we find the deviation of installation of CMG in vertical position:

$$\sigma = 43'30''.$$

Size of the CCD-matrix of the Canon EOS 450D CPU is $22.2 \times 14.8$ mm, thus we find the permissible deviation of installation of CCD-matrix in vertical position:

$$\varepsilon = 1°23'20''.$$

The accuracy of the horizontal levelling according to the technical characteristics is between $20''$ and $30''$, which does not exceed the permissible deviation of installation of the CMG and CCD-matrix in the vertical position.

MSE of coordinates measurement of the intersection images of the CMG is equal to half pixel in the image of $2.2 \mu$m.

$$m_{x_1} = m_{x_2} = \sqrt{m_{c_1}^2 + m_{c_2}^2},$$

$$m_{x_1} = m_{x_2} = 3.11 \mu m.$$

Accuracy of the CMG comparison is calculated.
MSE of the grid is taken $m_{c_1} = m_{c_2} = 2 \mu m$ [Kalantarov 1986], where the error of the segment between adjacent strokes is:

$$m_{x_1} = m_{x_2} = \sqrt{m_{c_1}^2 + m_{c_2}^2},$$

$$m_{x_1} = m_{x_2} = 2.83 \mu m.$$

Let us determine the accuracy of the micrometer. Distance of moving CMG was measured using a micrometer; therefore the total error of the micrometer screw would be:

$$m_k = \sqrt{2m_k'},$$

where:

$m_k'$ – counts on the micrometer scale.

Accuracy of construction micrometer strokes according to technical characteristics is 4 microns and

$$m_k = 5.6 \mu m.$$

Analysis of the obtained errors allows us to conclude that the equivalence of all parameters is true, since the tenth and hundredth parts of a micron are virtually insignificant, and rounding up to units of micron leads to equality of values. Consequently, MSE of determining of the focal distance of the DC will be presented as follows:

$$m_f = \sqrt{m_0^2 + m_0^* + m_k^2 + m_{x_1}^2 + m_{x_2}^2 + m_{x_1}^* + m_{x_2}^*};$$

$$m_f = 7.8 \mu m.$$

3. Conclusions

1. The methods for determining the focal length of DC have been analysed, which helped to understand the difficulties that arise when implementing the methods.
2. Method for determining the focal length of DC is proposed, that can subsequently be used to calibrate these cameras, and to perform topographic aerial survey with UAVs, which will improve the accuracy of determining the coordinates of terrain objects.
3. The a priori estimation of accuracy is calculated and errors are analysed, which influence accuracy of determination of the focal length of DC.
4. The purpose of further research is to improve the proposed method for determining the focal length of DC.
References


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