THE CALCULATION OF RECEIVED OPTICAL POWER DURING AIRBORNE LASER SCANNING OF WATER OBJECT

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Summary

In this paper calculation of some physical parameters of the receiver of laser scanning system is presented. Basic physical principles of laser scanning of water objects are considered. Ranging standard deviation for typical parameters of laser scanning system are calculated. Formula for calculation of reflected optical power for specular reflective objects is proposed.

Keywords

specular reflection • water surface • avalanche photodiode • signal to noise ratio • ranging standard deviation

1. Introduction

In recent years landscape mapping and solution of different applied problems airborne laser scanning is actively used. The main advantages of this method are high efficiency, accuracy, availability to implement the surveying in night time and quick obtaining of final results. The determination of point coordinates with high accuracy depends on following factors:

• surveying – fly height, beam divergence, location of the reflected point within the swathe,
• precision of determining of exterior orientation elements – GPS satellite geometry, distance to ground base station, IMU update frequency and resolution,
• atmosphere influence – aerosols availability and turbulence,
• technique of classification and data processing.

Application of airborne laser scanning for investigation of area with large number of water objects often does not give fine informational content results. Water has properties of reflection of laser beam. It is specular reflection of water surface and high absorption in near infrared wavelength (with this wavelength all commercial topographic airborne laser scanners work). Shortage of measured points in water surface and reducing them in wetlands leads to complication of the creation of digital surface model with high accuracy [Brzank and Heipke 2007].
The following reasons influence reducing value of laser pulse during transmission to object and return direction: absorption and scattering by the atmosphere, reflectivity of target, distance to the target and losses of receiving system. It should be noted, that atmosphere influence on reducing laser pulse twice as a laser beam transmit double way – from transmitter to the target and backward.

For investigation of water objects and mapping of bottom of shallow water basins (not deeper than 70 m) in last years bathymetric laser systems are actively used. Using two lasers with near infrared (1064 nm) and green (532 nm) wavelengths [Shan and Toth 2008] gives possibility to detect the depth, as green laser beam penetrates through water and reflects from the bottom. Such systems are more complicated and expensive and, therefore, less widespread. Besides, accuracy parameters of detecting coordinates of points on land are considerably worse than in topographic laser systems.

The main tasks of investigation are: to calculate value of reflected optical power from the water and land surface in different conditions; to determine possibility of receiver to detect received energy as reflected signal; to consider a question of determination of signal to noise ratio (SNR) of receiver.

2. Physical principles of laser scanning of water objects

If specular reflection occurs, only beams with angles of incident close to zero will come back to the receiver and can be fixed. For larger angles signal will be lost completely as a result of specular reflection. In real conditions any water surface is always rough and reflection occurs for larger angles too. This is because only some part of all area of illuminated surface can be located perpendicular to the beam (Figure 1). With increasing an angle of incident, the probability of such mutual disposition reduces. Therefore on the edges of scan strip we usually have very small part of measured points and they have very small intensity.

![Reflection of laser beam from the water](image)

*Fig. 1. Reflection of laser beam from the water in case of: a) horizontal surface, b) rough surface with $\alpha \neq \beta$, c) rough surface with $\beta = 0$*
In real conditions we can see two main cases – total absence of measured points on surface of water as a result of specular reflection and absorption (Figure 2a) or partial availability of points in places, where air carrier was flighting directly above basin (Figure 2b).

![Source: authors' study](image)

**Fig. 2.** Peculiarities of laser beams reflection from water surface: a) absence of measured points, b) partial availability of measured points

3. Calculation of the main parameters of the receiver

3.1. Calculation of reflected energy

For obtaining mathematical expression of reflected optical power $P_r$ from the water surface we use equation for diffuse reflected surface [Brenner and et. 2006]. When taking into account the losses in receiving optics, the level of reflected signal will be reduced. Therefore, we have to take into consideration the system transmission factor $\eta_{sys}$. Then we can write the equation:

$$P_r = \rho \cdot \frac{A_r \cdot P_t \cdot M^2}{2\pi R^2} \cdot \eta_{sys},$$  \hspace{1cm} (1)

where:

- $P_T$ – transmitted optical power,
- $\rho$ – reflectivity of target,
- $A_r$ – area of receiver aperture,
- $R$ – range from sensor to target,
- $M$ – atmospheric transmission factor,
- $\eta_{sys}$ – system transmission factor.
As we can see from the formula, the power of received pulse is directly proportional to the transmitted optical power and inversely proportional to the square of range from sensor to target.

In case of specular reflection laser light will be scattering not in solid angle of hemisphere $2\pi$, but in angle equal $\Omega = \left(\frac{\pi (2R \cdot \beta)^2}{4}\right) \cdot \frac{1}{R^2}$, where: $\frac{\pi (2R \cdot \beta)^2}{4}$ – the part of hemisphere area, in which reflected beam will be scattering; $\beta$ – the divergence of the laser beam in radians. From here $\Omega = \pi \cdot \beta^2$. For calculation of part of hemisphere area, in which reflected beam will be scattering, for large $R$, the value $D_{tar}$ can be neglected in comparison to $R \cdot \beta$ (Figure 3).

\[
\Omega = \left(\frac{\pi (2R \cdot \beta)^2}{4}\right) \cdot \frac{1}{R^2}
\]

Therefore formula $P_R$ becomes

\[
P_R = \rho \cdot \frac{A_r \cdot P_t \cdot M^2}{\pi \cdot R^2 \cdot \beta^2} \cdot \eta_{sys}
\]  

(2)

We use formula (2) to calculate the optical power of received pulse for typical system with parameters: $P_t = 2000$ W, $\eta_{sys} = 0.8$, $D_r = 0.1$ m ($D_r$ – the diameter of receiv-

Source: authors’ study

Fig. 3. Geometry of laser beam spreading that has been emitted by the laser rangefinder to the target and on its return direction (incidence angle 0°)
ing aperture), \( R = 1000 \) m, \( M = 0.8 \). As seen in previous researches [Бабушка 2012], for near infrared wavelength and small incidence angle the value \( \rho = 0.02 \). If considering the length of laser pulse equal 10 ns, then calculated value \( P_r \) will be presented in Joule: \( P_r = 1.7 \cdot 10^{-4} \) W = \( 1.7 \cdot 10^{-12} \) J = 1.7 pJ.

As we see, the calculated value of received optical power is considerably smaller in comparison to the transmitted optical power. But even this value will be possible only for ideally smooth water surface and for normal incidence angle (0°).

For comparison lets calculate the power of received pulse for diffusively reflected surface using formula (1), but reflectivity of target \( \rho \) will take equal 0.5. This value is more typical for natural surfaces (as vegetation). Calculated value will be presented in Joule too: \( P_r = 6.4 \cdot 10^{-7} \) W = \( 6.4 \cdot 10^{-15} \) J = 6.4 fJ.

As we can see, even with larger on more then one order value reflectivity, the calculated value of received optical power for diffusively reflected surface is almost in 1000 times less, than for specular surface. But it should be noted, that such value for the surface with diffuse reflection will be got with different angles of incidence, and for specular surface – only with normal angle of incidence.

Calculated values of received optical power are theoretical. In real conditions this values can be smaller, as a laser beam does not fall on perfect smooth surface.

### 3.2. Calculation of responsivity of optical detector

To detect the energy laser system has the optical receiver. The optical receiver is composed of a receiving optic, an optical detector and a transimpedance amplifier [Keiser 1983]. The optical detector is realized with a semiconductor photodiode that converts an optical signal into an electrical current. The photodiode parameters depend on the used laser wavelength. For wavelengths up to 1100 nm photodiodes made of silicon can be used. A primary criterion in the selection of an appropriate photodiode is its responsiveness which should be as high as possible and is calculated by the formula [Shan and Toth 2008]

\[
\mathcal{R}_0 = \frac{\eta \cdot q}{h \cdot \nu}, \tag{3}
\]

where:
- \( \eta \) – the quantum efficiency,
- \( q \) – the electron charge,
- \( h \) – the Planck’s constant,
- \( \nu \) – the frequency of a photon.

The quantum efficiency \( \eta \) is given by the selected photodiode, namely it depends on the material it is made of, and the wavelength of the detected light. For silicon photodiode and wavelength 1064 nm the quantum efficiency is equal 0.3. In this case value \( \mathcal{R}_0 = 0.257 \).
Often in laser scanning systems avalanche photodiode detectors (APD’s) are used. They have higher responsiveness, as a result of an internal amplification at the cost of avalanche multiplication $M$. For APD’s formula (3) becomes

$$\mathcal{R}_1 = \frac{n \cdot q}{h \cdot v} \cdot M$$

(4)

Avalanche multiplication factor for silicon photodiodes can be larger than 400. For calculation lets accept more typical value $M = 120$. Then: $\mathcal{R}_1 = 30.89$.

3.3. Calculation of the signal to noise ratio (SNR)

Taking into consideration different types of noises, especially background, dark and thermal noises, a situation can occur, in which the level of received signal will be as small as the threshold. In this case optical signal will be seen as a noise.

The main description that determines ability of receiver to detect laser pulse is the signal to noise ratio (SNR).

SNR can be calculated by the formula [Recadenbosch 1996]:

$$\text{SNR} = \frac{\mathcal{R}_0 \cdot M \cdot P \cdot G}{\sqrt{2q \cdot G^2 \left[ I_{ds} + F(M) \cdot M^2 \left( I_{db} + \mathcal{R}_0 \left[ P_r + P_{bg} \right] \right) \right] \cdot B + \sigma^2 B}}$$

(5)

where:
- $I_{ds}$ – APD’s surface dark current,
- $I_{db}$ – APD’s bulk dark current,
- $F(M)$ – excess photodiode noise factor,
- $P_{bg}$ – background power,
- $B$ – effective noise bandwidth,
- $G$ – gain of amplifier,
- $\sigma$ – equivalent input noise of amplifier (thermal noise).

Thermal noise will influence essentially only for very small values of received optical power. Input signal is amplified by the photodetector and amplifier. Thus thermal noise at the calculating is neglected. Note that in current systems the cooling of the amplifier is designed effectively and it minimizes the thermal noise.

The value of bulk dark current $I_{db}$ is smaller than the value of surface dark current $I_{ds}$. Therefore we can neglect it or make the brackets and add it to the surface dark current. The producers mainly present value of general dark current – $I_D$. Therefore formula (5) becomes

$$\text{SNR} = \frac{\mathcal{R}_0 \cdot P_r \cdot M}{\sqrt{2q \cdot \left( I_D + F(M) \cdot M^2 \cdot \mathcal{R}_0 \left[ P_r + P_{bg} \right] \right) \cdot B}}$$

(6)
Excess photodiode noise factor for APD’s can be obtained by the McIntyre formula [McIntyre 1966]

\[
F(M) = kM + \left( 2 - \frac{1}{M} \right) (1-k),
\]

(7)

where:

\( k \) – ionization ratio.

For PIN – photodiodes the value \( F(M) = 1 \), because \( M = 1 \). For APD’s the \( F(M) \) can take different values depending on many factors. Typical value \( F(M) \) for silicon photodiodes is variable from 3 to 7. For calculating we will take mean value \( k = 5 \).

Background power can obtained at the formula:

\[
P_{bg} = L_b(\lambda)A_r\Omega_r\Delta\lambda,
\]

(8)

where:

\( L_b(\lambda) \) – represents the spectral radiance of the sky background,

\( A_r \) – the receiver-system effective aperture,

\( \Omega_r \) – receiver-system acceptance solid angle,

\( d\lambda \) – the equivalent spectral width of the spectrally selecting component.

For calculation of background power, we use value \( L_b(\lambda) \) for water surface and wavelength 1064 nm from literature [Wolfe and Zissis 1989] equal 20 W · m^\(-2\) · μm^\(-1\) · sr^\(-1\) and \( d\lambda = 10 \) nm. Then \( P_{bg} = 12.3 \cdot 10^{-15} \) W.

For the calculation we will take value \( I_d \) for photodiode of Perkin Elmer Canada Inc., model C30954E equal 1 · 10^\(-7\) A. This photodiode can work as PIN and APD mode.

The received photocurrent after detecting by the photodiode must be converted into a voltage. This is accomplished by a transimpedance amplifier.

Let us calculate the value SNR for PIN and APD photodiodes for specular and diffuse reflection at the formula (6), considering \( B = 1/T_p \) and \( P_r \) as calculated in 2.1.

For comparison, we substituted into the formula the values \( P_r \) in 10 and 100 times lower than value in 2.1 for diffuse reflection. The results are presented in Table 1.

**Table 1.** The calculated SNR for different values of received optical power

<table>
<thead>
<tr>
<th>( P_r, W )</th>
<th>( SNR ) for ( PIN )</th>
<th>( SNR ) for ( PIN ) in dB</th>
<th>( SNR ) for APD</th>
<th>( SNR ) for APD in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7 · 10(^{-4})</td>
<td>1166</td>
<td>61</td>
<td>522</td>
<td>54</td>
</tr>
<tr>
<td>6.4 · 10(^{-7})</td>
<td>56</td>
<td>35</td>
<td>32</td>
<td>30</td>
</tr>
<tr>
<td>6.4 · 10(^{-8})</td>
<td>8.5</td>
<td>18</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>6.4 · 10(^{-9})</td>
<td>1</td>
<td>0</td>
<td>3.2</td>
<td>10</td>
</tr>
</tbody>
</table>
As we see, when sufficient level of received optical power the PIN – photodiodes allow to get higher SNR. APD’s are more acceptable for very small values of received optical power.

Considering SNR the ranging standard deviation $\sigma_R$ for pulses with perfect rectangular shape with width $T_p$ can be obtained in the formula [Shan and Toth 2008]:

$$\sigma_R = \frac{c}{2\sqrt{2}} \cdot \frac{1}{\sqrt{\text{SNR}}} \cdot T_p,$$

where:

$c$ – speed of light.

Calculated values $\sigma_R$ are presented in Table 2.

<table>
<thead>
<tr>
<th>SNR</th>
<th>$\sigma_R$, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1166</td>
<td>3.1</td>
</tr>
<tr>
<td>522</td>
<td>4.6</td>
</tr>
<tr>
<td>56</td>
<td>14.1</td>
</tr>
<tr>
<td>32</td>
<td>18.7</td>
</tr>
<tr>
<td>10</td>
<td>33.3</td>
</tr>
<tr>
<td>8.5</td>
<td>36.3</td>
</tr>
<tr>
<td>3.2</td>
<td>59.2</td>
</tr>
</tbody>
</table>

Producers of the equipment declare ranging standard deviation 1–2 cm for ideal conditions. In last years there is a tendency for reduction output optical power of laser. It’s leads for reduction of SNR. Therefore, it is nearly impossible to achieve such accuracy in real conditions.

4. Conclusions

1. The main losses of signal by scanning water objects are caused by specular reflection of beam from the water surface; absorption of electromagnetic energy near infrared wavelength by water has smaller influence.

2. When considering specular reflection of water objects, the laser scan angle during surveying must be as small as possible.

3. When implementation of laser scanning of water objects it is more expedient to use avalanche photodiode as detector. This can essentially raise informational content of laser scanning data.
4. Photodiode dark current does not have essential influence on SNR of a receiver.
5. The main factor influenced on SNR of receiver is the level of received optical power, which in turn depends on the output power of laser and distance to the object.
6. In real conditions it is difficult to rich the value of ranging standard deviation declared by producers.

References


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