

Assessment of spatial distribution of the geodetic vertical network points – a case study of Wieliczka County

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Summary

Access to geodetic control network points is key to the success of most surveying works. Particular attention should be paid to designing the locations of geodetic vertical control network points, as they enable the correlation of measurements and the implementation of a valid vertical frame in a given area. The development of GIS tools has made it possible to analyse the density of geodetic networks and their spatial relationships with the surrounding area. This article applies classical analysis methods (Voronoi diagrams, nearest neighbour analysis, distance matrix) to examine the spatial distribution of a geodetic control network in a selected county. The study focused on the detailed vertical control network that was modernized in 2021, as well as the existing points of the basic vertical geodetic network in the Wieliczka County. The analyses showed that despite the destruction of 48% of the points of the basic vertical geodetic network, the configuration of remaining points can still be effectively used to establish a detailed vertical control network. The density of the existing vertical network in Wieliczka County averages 1.6 points/2 km². Approximately 83% of buildings have two vertical points within a 1 km radius. The distances the existing vertical geodetic network points (basic and detailed) indicate the economic feasibility of establishing a height measurement network in the vast majority of Wieliczka County (for approximately 57% of the area, the sum of straight-line distances does not exceed 1.5 km). Research also revealed inconsistencies between publicly available databases of geodetic networks.

Keywords

geodetic vertical network • geodetic network density • levelling line length • geodetic network spatial distribution

1. Introduction

The development of precise positioning methods and techniques using the Global Navigation Satellite System (GNSS) has streamlined and accelerated surveying. Relative positioning methods, such as Real Time Kinematic (RTK) and Network Real Time Kinematic (NRTK), have found particular application in surveying. However, relative positioning requires the development of appropriate supporting infrastructure, which in this case consists of a dense network of ground reference stations continuously recording GNSS signals, along with a computing centre. European countries have developed national reference networks based on the European Position Determination System (EUPOS) standards, e.g., ASG-EUPOS (Poland), CZEPOS (Czech Republic), SKPOS (Slovakia). EUPOS standards specify that the average distance between stations should not exceed 75 km. For example, the ASG-EUPOS network has a density of around 70 km [Kudas 2020], which optimizes the vector lengths for postprocessing static observations [Kudas et al. 2022] and precisely determines the heights using NRTK techniques [Kudas and Wnęk 2022]. Most surveying work has begun to move away from classical measurement methods based on physical geodetic control network points in the field. However, there is still a range of surveying work that requires the use of classical geodetic control networks, particularly for height measurements. As declared by administrators of real-time positioning services in Poland [Geoforum 2020], and confirmed by numerous studies [e.g., Próchniewicz et al. 2020, Kudas et al. 2017, Kudas and Wnęk 2020, Uznański 2023], existing services provide an accuracy of ± 0.03 m for horizontal coordinates and ± 0.05 m for height. The accuracy of determining horizontal coordinates using the NRTK technique meets the accuracy requirements for measuring the position of situational details set by the technical standards for performing geodetic measurements in force in Poland [Regulation 2020]. However, the declared height measurement accuracy is insufficient to determine the height of some technical infrastructure facilities. Such objects include elements of the sewage network where gravity flow is involved. Accurate determination of the planned heights and slopes in the field and performance of post-construction inventory measurements require the use of conventional measurements based on a height control network. Additionally, due to the possibility of irregularities in the initialization of the GNSS receiver and the difficulty in detecting this phenomenon during measurements, it is recommended to perform control measurements at points with coordinates known with appropriate accuracy, i.e. geodetic control network points.

Therefore, there is still a need to implement and maintain a functional geodetic control network with an appropriate degree of density, especially a vertical control network. Spatial analyses of geodetic control networks primarily focus on horizontal control networks [e.g., Oleniacz and Skrzypczak 2012, Bielecka et al. 2014, Calka et al. 2017, Kudas 2020], which means there is a lack of no analyses devoted to vertical control networks. The aim of this study was to use spatial analyses using the free QGIS software to assess the density and availability of vertical control networks in a selected county. Additionally, it was examined whether there was a statistically

significant relationship between the spatial distribution of vertical control points in the analysed area.

2. Legal regulations regarding the geodetic vertical network in Poland

The geodetic vertical network in Poland was shaped by the provisions of the no longer applicable Regulation of the Minister of Administration and Digitization of February 14, 2012, on geodetic, gravimetric and magnetic control networks (Journal of Laws of 2012, item 352) and Technical Instructions G-1.11 (Basic Vertical Geodetic Network) and G-2 (Geodetic Vertical Control Network). Currently, the geodetic control network in Poland operates based on legal regulations specified in 2021, by the Regulation of the Minister of Development, Labour and Technology of 6 July 2021 on geodetic, gravimetric and magnetic control networks (Journal of Laws of 2021, item 1341). The Regulation [2021] specifies that the basic geodetic network is divided into fundamental (first-order) and base (second-order) classes in terms of accuracy, and its task is to transfer the European reference system to the territory of Poland, according to the type of network. The fundamental class of the basic vertical geodetic network is formed by the nodal points of the national part of the European levelling network, and the base class is formed by the points determined by the precise geometric levelling technique or the satellite levelling method, as well as the points of the national part of the European levelling network not included in the fundamental class [Regulation 2021]. The fundamental and base classes of the basic vertical geodetic network are characterised by an accuracy of no less than 0.01 m in determining point height.

In Poland, there is also a detailed vertical control network, which consists of points related to the basic geodetic network. A detailed vertical control network is a collection of points that serve to densify the basic geodetic network. It is used to establish measurement networks and perform geodetic measurements. The density of detailed control network points depends on the demands arising from the economic development of a given area. The density of detailed vertical control network points, including those from the existing basic geodetic network, should at least 1 point/2 km² in areas of existing or planned dense development, and 1 point/5 km² in other areas [Regulation 2021]. A detailed vertical control network is created by levelling networks established using geometric levelling, whose structural elements are levelling lines consisting of levelling sections or vertical network points established through satellite levelling. The later is linked to a minimum of four points of the basic vertical geodetic network [Regulation 2021]. Detailed vertical control network points are permanently stabilised (using ground, wall, or structure markers), and their height determination error should not exceed 0.01 m relative to the basic vertical geodetic network. The provisions of the Regulation [2021], like the previous Regulation [2012], specify that the length of levelling lines when establishing a detailed control network should not exceed 18 km, and 6 km in urban areas. Levelling lines should create levelling sections ranging from 0.5 km to 1.0 km in length, and in non-urban areas, these sections should not exceed 5 km. The repealed regulation allowed levelling sections up to 3 km long in non-urbanised

areas, with the possibility of increasing it to 4.5 km if it was impossible to install wall markings due to the lack of buildings in the area [Regulation 2012].

The database containing information on the fundamental and base classes of the basic geodetic network in Poland, known as the National Register of Basic Geodetic, Gravimetric and Magnetic Networks (PRPOG), is maintained by the Head Office of Geodesy and Cartography (GUGiK). Meanwhile in the case of the detailed control network, The Detailed Control Network Database (BDSOG) is maintained by the heads of counties through the county surveyor in charge of the County Centre for Geodetic and Cartographic Documentation (PODGİK).

3. Materials and methods

The research used publicly available information on the detailed vertical control network made available via the PODGiK portal in Wieliczka (<https://wielicki.webe-wid.pl/e-uslugi/portal-mapowy>) and data on the basic vertical geodetic network made available via the WMS service of the national geoportal (<https://www.geoportal.gov.pl/>). The analyses were carried out in the PL-2000/21 (EPSG 2178) horizontal coordinate system, dedicated in Poland for large-scale studies. The research used also data on buildings from the land and building register database (EGiB) and the state register of administrative unit boundaries (PRG) available to download via the national geoportal (<https://www.geoportal.gov.pl>). All spatial analyses and calculations were performed using QGIS v3.40.7.

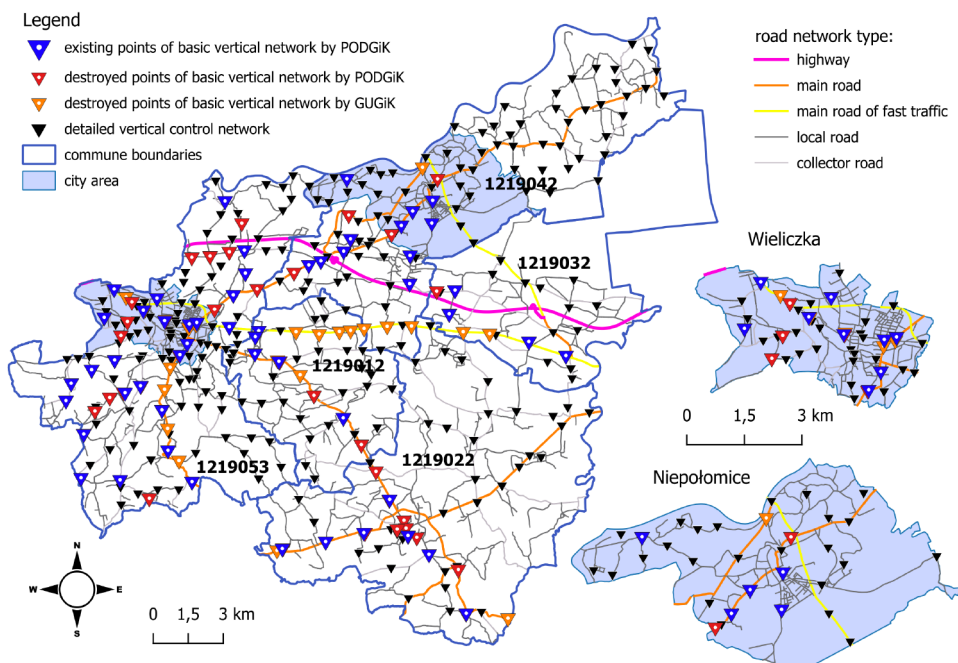
4. Vertical network in the Wieliczka County

Wieliczka County [TERYT 1219] is located south of the city of Krakow and belongs to the commuting zone of Krakow metropolitan area. The ellipsoidal area of Wieliczka County is 410.5987 km². Wieliczka County is divided into two urban-rural communes (Wieliczka (1219053) and Niepołomice (1219043)) and three rural communes (Biskupice (1219012), Kłaj (1219032), and Gdów (1219022)). The county includes two cities (Wieliczka, Niepołomice).

A detailed vertical control network in the Wieliczka County was developed in the 1970s and modernised in 2021. The network modernisation was based on a 2019 technical design (Agreement No. GK.272.16.2019 of 11 July 2019), which was prepared by a surveyor authorised for Scope No. 3 – Basic Surveys. As noted in the modernisation project [2019], changes to the development and modernisation of buildings that occurred after the former vertical network was established have contributed to the destruction of a significant number of vertical benchmarks. The project therefore included a levelling survey of the entire planned vertical network, while simultaneously inventorying the existing vertical network points and establishing new benchmarks to densify the vertical network. During the inventory, approximately 50% of the benchmarks were found to be destroyed or damaged, mainly due to the widespread thermal insulation of buildings. Additionally, the project involved adapting monitoring bench-

marks that meet the technical requirements and belong to the monitoring network of the mining areas of the Wieliczka Salt Mine, and do not show vertical movements during documented cyclic measurements. The project also excluded points located in the areas prone to frequent landslides in the Wieliczka County.

New benchmarks were also designed for areas of the county where no vertical control network had previously been established (Kłaj, Targowisko, Cichawa, Niegowić, and Szczytniki cadastral districts). However, the design of points south of the Raba River and in forest areas was dropped for economic reasons and due to low urbanisation. To ensure access to the newly established benchmarks, the design focused primarily on bridges, permanent culverts, and retaining walls. In the absence of appropriate development, ground markers were used instead. The project assumed the presence of levelling lines up to 1 km long in built-up areas and up to 2 km in other areas. The modernisation project [2019] assumed that the modernised vertical control network would have a total of 333 points, including 271 detailed control points and 62 basic vertical geodetic points. The 2019 modernisation project assumed the adaptation of the existing 112 lower-class benchmarks and the installation of 157 new wall benchmarks and 2 earth benchmarks. The planned total length of the network's levelling lines was to be 448 km (average line length 1.8 km).



Source: Author's own study

Fig. 1. Location of the base class points of the basic vertical geodetic network and the detailed vertical control network in the area of Wieliczka County and the cities of Wieliczka and Niepołomice against the background of the main communication routes

In Wieliczka County, there is 1 point in the fundamental class of the basic geodetic network and 102 points of the base-class points in the basic vertical geodetic network, of which only 77 points can be described as in good condition, while the remaining points are damaged or destroyed [Register of geodetic networks 2023]. However, data provided by Wieliczka County PODGiK portal indicates that only 53 points of the basic vertical geodetic network were found in the field in 2025. The data set regarding the detailed vertical control network contains information on the existence of 273 points, including 1 destroyed point. Therefore, in Wieliczka County, there are a total of 323 points of the basic and detailed vertical network (Fig. 1). In 2025, a tender was also concluded for the modernisation of the basic vertical geodetic network in the Małopolska Voivodeship (including lines No. 332 and No. 370 running through Wieliczka County) [gov.pl 2025].

5. Spatial analyses of vertical network accessibility

The availability of vertical control points in a given area can be reduced to measurable parameters, such as the unit of area per control point or the minimum distance from a given spatial point to a specified number of vertical control points (k).

A classic way to assess the spatial pattern of a selected point phenomenon is nearest neighbour analysis (NNA), a statistical method used to understand the distribution of spatial data and determine whether points in a given coordinate system are randomly distributed, clustered, or uniformly distributed. The NNA result is the nearest neighbour index (NNI), which indicates clustering for $NNI < 1$, random distribution for $NNI = 1$, and uniform/scattered distribution for $NNI > 1$. The NNI index can be determined using the following formula (1) [PQStat 2025]:

$$NNI = \frac{\overline{NN}}{\overline{d_{exp}}} \quad (1)$$

where:

$\overline{NN}$ – mean observed nearest neighbour distance,
 $\overline{d_{exp}}$ – mean expected nearest neighbour distance.

The above statistics used to determine NNI are calculated according to formulas (2) and (3):

$$\overline{NN} = \frac{\sum_{i=1}^n d_i(NN)}{n} \quad (2)$$

$$\overline{d_{exp}} = \frac{1}{2} \sqrt{\frac{A}{n}} \quad (3)$$

where:

$d_i(NN)$ – the distance of each point from its nearest neighbour ($k = 1$) defined as the Euclidean metric,

- n – the number of points,
 A – the area of the region.

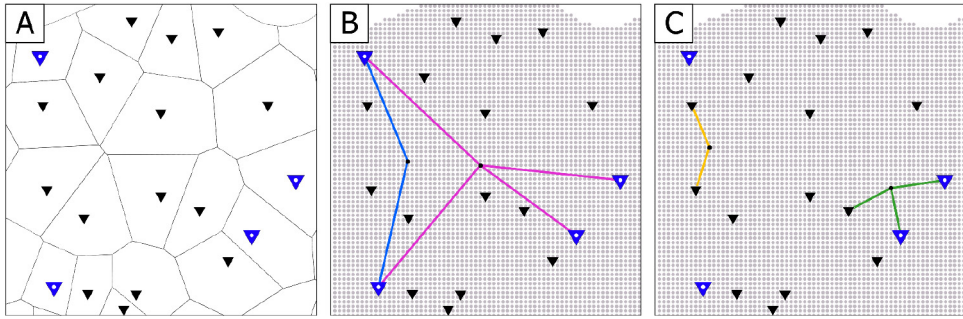
The geometric plane can be successfully divided based on the locations of point objects using a Voronoi diagram (also known as Dirichlet tessellation). This simple method of dividing geometric planes has been used for years in analyses of the distribution and accessibility of socially significant objects [Kisiała and Rudkiewicz 2017] and in studies of the spatial pattern of geodetic networks [Calka et al. 2017, Kudas et al. 2020, Bespalko et al. 2022]. The Voronoi diagram for a set of points P is a division of the plane into n Voronoi cells $V(p_1), V(p_2), \dots, V(p_n)$. A Voronoi cell is a polygon constructed around a given point such that any point within the polygon is closer to that point than to any other point in the analysed set. For a given polygon V drawn around any point from the set $P\{p_1, p_2, \dots, p_n\}$, where $p_i \in \mathbb{R}^2$ and $n \in \mathbb{N}$, this relation can be determined using the following formula [4] and the Euclidean metric denoted as d [Okabe et al. 2009]:

$$\forall(p_i) = \{x \in \mathbb{R}^2 \mid \forall_{j \neq i} d(x, p_i) \leq d(x, p_j)\} \quad (4)$$

where:

- $d(x, p_i)$ – distance of point x belonging to the Voronoi cell created around point p_i to point p_i ,
 $d(x, p_j)$ – distance of point x belonging to the Voronoi cell created around point p_i to point p_j around which another Voronoi cell was created.

When establishing a detailed vertical control network using geometric levelling, it is worth considering the distance to the 2 nearest points of the baseline control network and, when using satellite levelling with GNSS, to the 4 nearest points of the baseline control network. Satellite levelling requires the determination of spatial vectors to the nearest points of the baseline network. When performing height measurements, it is necessary to reference the 2 nearest points of the baseline or detailed vertical control network, as this is the minimum number of points required to establish a levelling line and a measurement network. The distance to the three nearest control network points can also be considered, as geodetic measurements require additional observations for correct error estimation. In a simplified GIS analysis, these distances can be determined as straight-line distances between a given point and the reference points. In reality, levelling lines should be drawn to account for terrain features and terrain obstacles to ensure proper visibility. Due to the above, the levelling lines implemented in the field are not straight lines, but lines with numerous bends resulting from the location of the surveyor's level in development and topography of the terrain. Determining the distance from a given point to the k nearest control network points can be performed in GIS programs using a distance matrix. For this purpose, the research used a discrete model of the Wieliczka County in the form of a regular grid with dimensions of 50×50 m, which gives a total of 164,061 points. The grid points were assigned and then the distance to $k \in \{2, 3, 4\}$ the nearest points of the vertical control network.



Source: Author's own study

Fig. 2. Diagrams of the considered accessibility to the vertical control network: Voronoi diagram (A), sum of straight-line distances to the 2 or 4 nearest points of the base vertical control network from the grid model (B), sum of straight-line distances to the 2 or 3 nearest points of the vertical control network from the grid model (C)

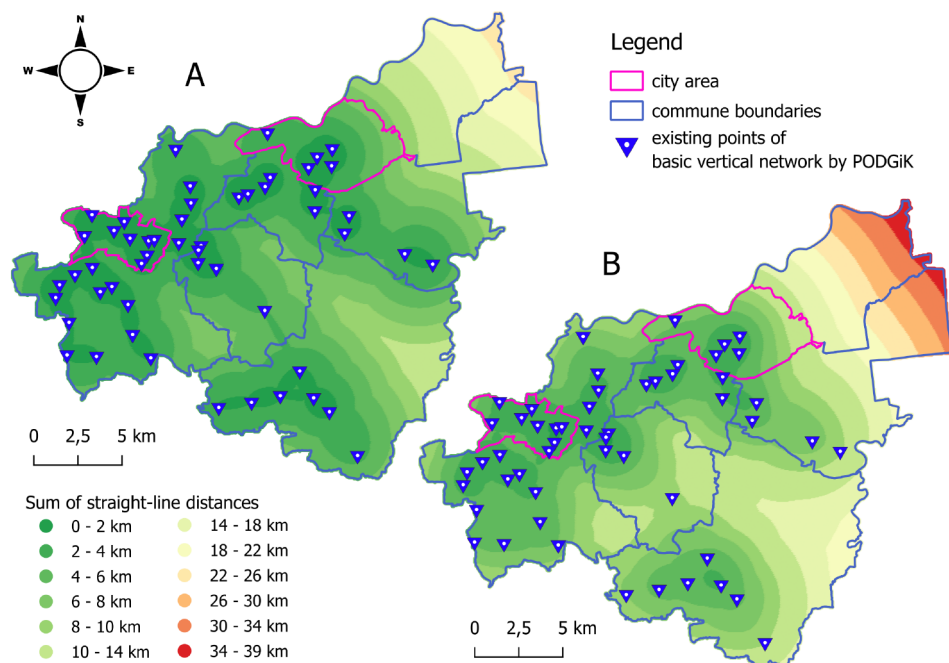
6. Results and discussion

The nearest neighbour analysis for all vertical control points in Wieliczka County showed that the observed average distance is 0.6 km ($\overline{d_{exp}}$ 0.7 km), and the NNI is equal to 0.925 (Z-Score -2.591). However, the NNI for the 53 points of the basic vertical geodetic network alone was 0.867 (Z-Score -1.857), with an observed average distance of 1.2 km and ($\overline{d_{exp}}$ equal to 1.4 km). Therefore, for all points of the vertical control network, the existence of significant clustering can be stated, while for the basic vertical geodetic network itself the result does not provide strong confirmation of clustering at $\alpha = 0.05$. For comparison, according to the study by Oleniacz and Skrzypczak [2012], in the selected 15% of counties in Poland, the former class III horizontal control network was characterised by an average distance of points of about 0.250 km.

The average length of the tie to the two nearest points from the set of existing 53 base network points is 4.9 km, while to the four nearest base network points is 7.4 km. In the case of tie-in to two base network points, the length of the sum of straight-line distances does not exceed 6 km for approximately 67.8% of the county's territory, and the length of the sum of straight-line distances in the range of 6 to 10 km covers 16.7% of the county's territory, while values in the range of 10 to 18 km characterise the tie-in covering 10.6% of the county's territory (Fig. 3). However, it should be remembered that the final accuracy of the height of the points of the established vertical network will also be influenced by the shape of the network, the quality of observation data and the method of network alignment, as noted by Cvetkov (2024), and not only by the length of the levelling lines to the points of known height.

In the case of establishing a detailed vertical control network for 95.1% of the county's territory, the length of the sum of straight-line distances to the two nearest base control points will not exceed 18 km (Fig. 3). In the case of connecting to four base control points, the sum of straight-line distances will not exceed 6 km in 30.7% of

the county's territory, while the sum of straight-line distances will not exceed 18 km in 59.2% of the county's territory. The sum of straight-line distances will not exceed 4 km in length when connected to only the two nearest base control points in 47.5% of the county's territory and in 11.6% of the county's territory, when connected to four base control points. Given the destruction of a significant number of base vertical control points in Wieliczka County, modernisation of the basic vertical geodetic network in the central part of Wieliczka County may be necessary for the future establishment of new detailed vertical control points [gov.pl 2025].

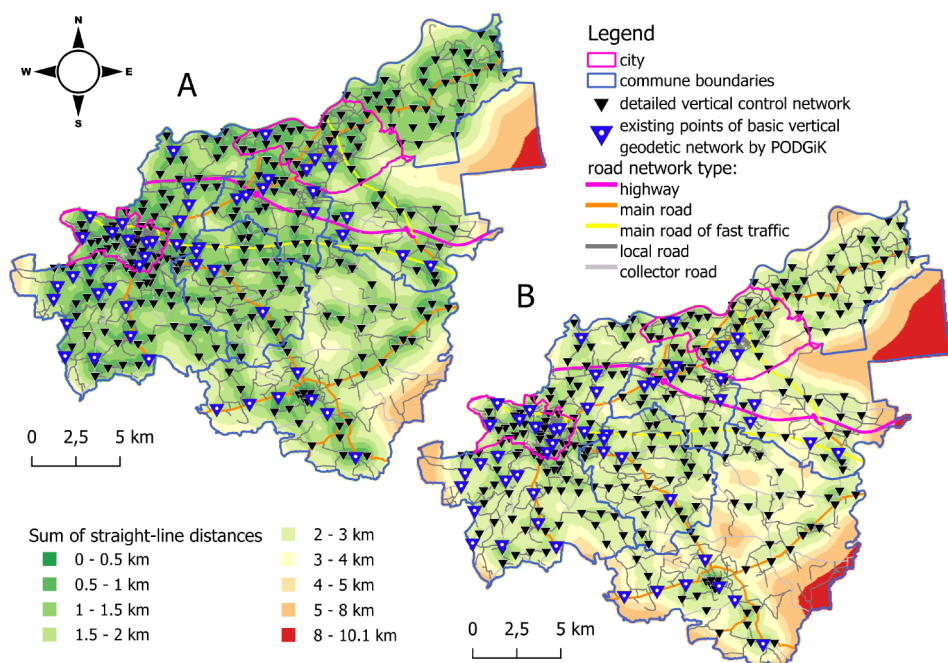


Source: Author's own study

Fig. 3. Sum of straight-line distances to the two nearest points of the basic vertical geodetic network (A) and to the four nearest points of the basic vertical geodetic network in Wieliczka County (B)

A measurement network should be established based on a minimum of two points of a basic or detailed vertical network when using geometric levelling, trigonometric levelling, and satellite levelling using GNSS static technology [Regulation 2021]. The analysis showed that approximately 57.0% of the county's area is located in such a relationship to the existing two vertical control network points that it allows for measurement when implementing a sum of straight-line distances no longer than 1.5 km (Fig. 4A). Therefore, under optimal conditions, a minimum of 15 geometric levelling stations using the centre-based method is sufficient to establish a measurement vertical

network with a mean error of no more than ± 0.02 m. This approach provides a much more accurate height determination than using RTK/NRTK measurements, for which an accuracy of ± 0.05 m is assumed [Geoforum 2020]. However, as noted by Michal and Štroner [2024], correct measurement implementation requires more than just the proximity of network points; the configuration of network points and the expected accuracy are also important. For references to the three nearest vertical control points, the sum of straight-line distances will not exceed 1.5 km in approximately 9.5% of the territory of the Wieliczka County (Fig. 4B).

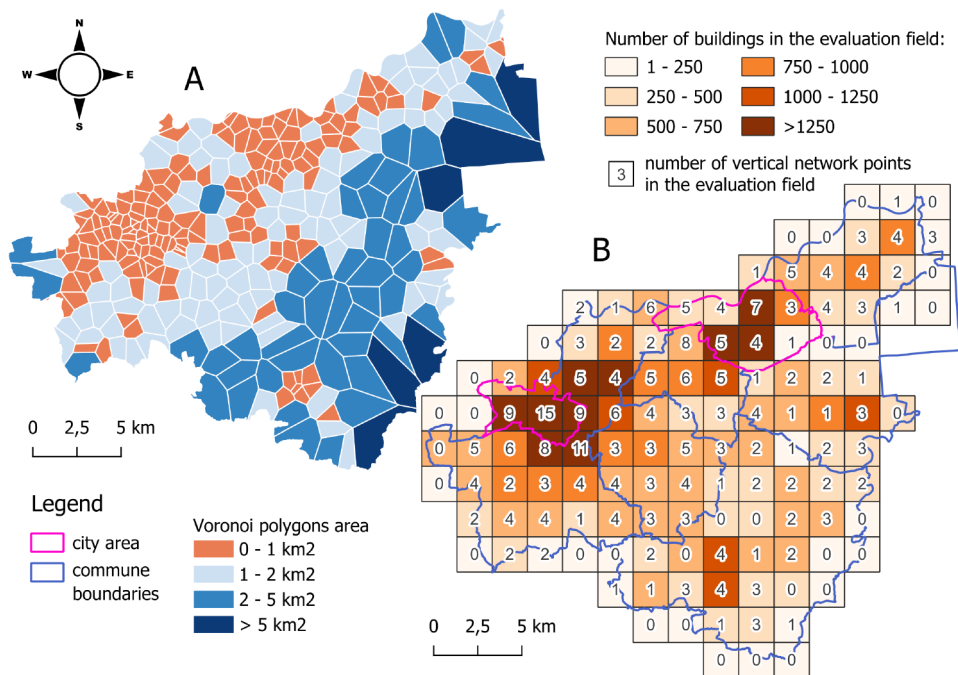


Source: Author's own study

Fig. 4. Sum of straight-line distances to the two nearest points of the basic or detailed vertical network (A) and to the three nearest points of the basic or detailed vertical network (B)

The areas where sum of straight-line distances exceeding 3 km in length would be necessary to connect the assumed measurement network points to the two or three nearest vertical control points are not significant in terms of the total area of Wieliczka County. These are areas concentrated in the eastern part of the analysed area (Fig. 4). If the assumed measurement network points are connected to the two nearest vertical control points, the length of the sum of straight-line distances will exceed 3 km in 10.9% of the considered area, and if it is necessary to connect them to the three nearest vertical control points, in 33.9% of the Wieliczka County total area.

Analysis of the vertical control network in Wieliczka County using the Voronoi diagram shows a regular change in the density of the vertical control network in the form of SW-NE strips, decreasing with distance from the city limits of Krakow (Fig. 5). The strip with the highest network density, i.e., 1 point/1 km², extends along the Wieliczka-Niepołomice line.



Source: Author's own study

Fig. 5. Division of the Wieliczka County area into Voronoi polygons in relation to the points of the basic and detailed vertical network (A) and the number of the points of the basic and detailed vertical network against the background of the building number in the evaluation field of 4 km² in the Wieliczka County (B)

In the case of correlation analysis of the existing number of vertical network points and buildings in the square-shaped assessment fields (1 km², 2 km², and 4 km²), a positive correlation was observed, indicating that as the number of buildings increases, so does the number of vertical network points (Fig. 5B). In the case of correlation analysis in the 1 km² and 2 km² assessment fields, the correlation coefficient was 0.55, and in the case of the 4 km² assessment field, it was 0.80, indicating moderate and strong correlations, respectively. Therefore, it can be concluded that the control network is well-matched to the existing development. The existing vertical control network replicates the spatial pattern of building clusters, which is related to the dominance of

wall-mounted vertical markers in the network stabilisation method. The average total distance of existing buildings in Wieliczka County to the nearest two vertical control network points is approximately 1.2 km. Approximately 16,800 buildings have two vertical control network points within a 0.5 km radius of their centroid, and approximately 58,300 buildings have two within a 1 km radius. However, only approximately 0.8 thousand buildings have a distance exceeding 2 km to the nearest single vertical control network point. As AbdAllah and Wang [2022] notice geodetic network is designed according to precision, reliability, and cost criteria, which translates into the optimisation of the number of points and their location according to the demand in the area. This can be observed in the case of the analysed network and its fitting into the development. The obtained results prove that it is possible to densify the geodetic vertical control network by establishing a measurement network using geometric levelling in the case of sewage installation measurements and achieving better height measurement accuracy than ± 0.05 m, which is obtained in the case of using GNSS measurements and converting ellipsoidal heights to normal heights in the PL-EVRF2007-NH system using the valid quasi-geoid model PL-geoid-2021 [Trojanowicz and Owczarek-Wesołowska 2023].

7. Conclusions

The density of the existing vertical control network in Wieliczka County averages 1.6 points/2 km², exceeding the mandatory value of 1 point/2 km² specified in applicable law for existing or planned dense development by 60%. This facilitates the use of the vertical control network for surveying purposes and enhances its reliability. Analyses have shown that approximately 83% of buildings have two vertical control points within a 1 km radius. The vertical control network is well-suited to land development in Wieliczka County, as it is located in close proximity to developed areas and allows for the implementation of the national vertical system PL-EVRF2007-NH for surveying purposes. Nearest neighbourhood analysis revealed a statistical relationship between the spatial distribution of vertical control points in the analysed area – the studied points do not follow a random spatial distribution, but are concentrated primarily in developed areas. Additionally, approximately 67.8% of the county's area is located in a relationship to two points of the basic vertical geodetic network that allows for the construction of sum of straight-line distances no longer than 6 km for the purpose of establishing a detailed vertical control network. Therefore, despite the destruction of approximately 48% of the basic vertical geodetic network in Wieliczka County, the preserved points still allow for effective network compaction over more than half of the county. If it is necessary to establish a measurement vertical control network using geometric levelling, the length of the sum of straight-line distances to the two nearest points of the basic or detailed vertical network will not exceed 1.5 km in 57% of the Wieliczka County area and 2 km in 75% of the Wieliczka County area.

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