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MATHEMATICAL-PHYSICAL MODEL OF HORIZONTAL REFRACTION IN MEASURING ALIGNMENT OF ELONGATED ENGINEERING OBJECTS

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Summary

While performing geodesic tasks of highest fidelity in machinery production plants, ironworks, rolling mills etc. – including monitoring of the geometry of production lines, geodesic measurements of dislocations and deformations of gravity dams and many other kinds of geodesic tasks, laser alignmetric methods have been used for decades. An important disadvantage of these simple optical methods lies in the negative impact of geodetic refraction on the path of the laser beam. This paper presents a theoretical mathematical-physical model for calculating refraction adjustment (line corrections) including a sample calculation.

Keywords

alignment method • refraction • temperature gradient • refractive adjustment

1. Introduction

Horizontal refraction is the deflection of laser beam from its straight path, seen in either horizontal or vertical projection. Theoretical and experimental research into how the phenomenon of refraction impacts geodetic measurements led to further clarification of mathematical formulae describing the path of the refraction [Bryś 1996]. In the environment where the measurement is conducted – near water courses and bodies of water (rivers, lakes), in densely built industrial zones, in the vicinity of technical buildings and structures – variability of air temperature is considerable, causing a refraction field which is variable in time and space, and which generates deviations of the optical radius and laser point even up to several centimetres at a distance of 80–100 metres. This has been confirmed by numerous experimental research projects conducted by geodesists over the last 50 years (Bryś, Beluch, Bendemann, Chrzanowski, Ingensand, Ostrovskij, Trevogo, Mosschuchin, Hennes, Bahnert, Brunner, Deumlich, Johnston, Korittke, Moritz, Heister, Schwarz, Wilhelm, Witt and other researchers [Ostrovskij 1958, Moritz 1962, Werner 1985, Holejko 1987, Bryś 1995, Hennes 1998, Johnston 1991], http://www.derwesten.de/ staedte/ /nachrichten-aus-wi/).

Refraction is a physical phenomenon known since the dawn of mankind (for instance, Fata Morgana mirage, mirror reflections of ships off the surface of the sea, mirror images of cars above the surface of a heated asphalt road, hot air rippling over a field of corn, elliptical shape of the sun over the horizon just before the sunset, etc.). Theoretical and experimental scientific research into the phenomenon goes back to the second half of the 19th century (see Bendemann – Prussian triangulation).

Presently, it is customary to subdivide geodesic refraction into:

- astronomical/atmospheric refraction (change in the direction of the propagation of E-H radiation in Earth's atmosphere),
- spatial/cosmic refraction (change in the direction of the light radiating from farther stars, for instance, the surface and corona of the Sun, the Moon),
- ionospheric refraction (85–200 kilometres above sea level),
- ground level/tropospheric refraction (up to 10–13 kilometres above sea level – depending on the latitude on the globe),
- trigonometric refraction,
- geometrical levelling refraction,
- mining refraction (variable location of the laser beam in different seasons and hours of the day; also in different wind speeds, varying ventilation of excavations and shafts, etc.),
- electro-optical refraction (in the environment of a plane-parallel field around DC wires) [Bryś 2012].

When light beams are crossing objects of varying density, and in variable meteorological conditions, the following five interconnected physical effects occur [Hennes 1998, Bryś 2012]:

- extinction (decrease in the density of light, and therefore also in its range),
- diffraction (deflection of the light beam in the immediate vicinity of barriers in the terrain),
- change in the horizontal and vertical direction of the laser target – GEODETIC REFRACTION,
- change in the propagation of light beam and related effects (electrical measurement of distance, etc.),
- short-period, chaotic change of light beam location (e.g. blinking of the laser point).

2. Basic mathematical and physical formulae

Mathematical foundations when developing theoretical models and sophisticated refraction solutions for precise measurement issues in a variable meteorological and/or microclimatic field are provided by the following fundamental formulae [Bryś 1995, 1996], [Ostrovskij 1958]:

$$\delta_A = \delta_B \tag{1}$$

$$\delta = \frac{2}{S} \int_A^B \nabla_x n \cdot (S - S_i) ds \tag{2}$$

and

$$\nabla_h^{\acute{s}r} n = \frac{\partial n}{\partial T} \cdot \frac{\partial T}{\partial h} = (n_o - 1) \cdot \frac{T_o}{T^2} \cdot \frac{p}{p_o} \cdot \frac{\partial T}{\partial h} \tag{3}$$

Symbols in the formulae and the drawings:

$\delta_A = \delta_B$ – partial horizontal refraction angle (Figures 1 and 2),

δ – full horizontal refraction angle,

S – length of the alignment base (Figure 2),

$\nabla_x n$ – gradient of the atmospheric refraction index for specific meteorological parameters [Bryś 1994],

n – atmospheric refraction index during measurement (T, p, e),

n_o – atmospheric refraction index for standard atmosphere (T_o, p_o, e),

τ_x – horizontal atmospheric temperature gradient in the environment of the alignmetric base,

λ – length of the light beam or the laser beam,

Δh – width of the laser beam.
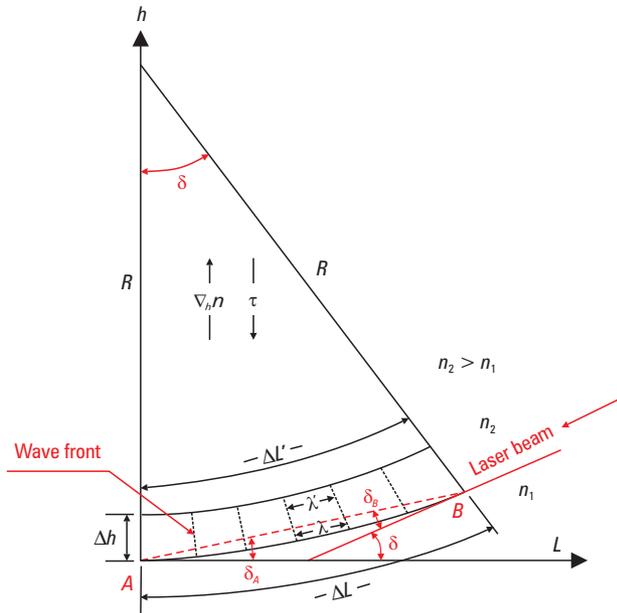
Source: authors' study

Fig. 1. Physical-geometrical interpretation of the laser beam geometry within the refraction field
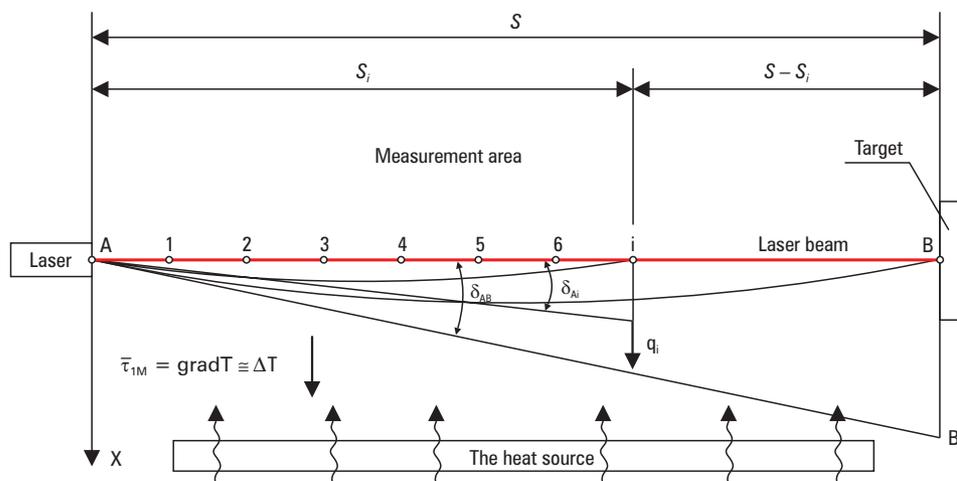
Source: authors' study

Fig. 2. Graphic illustration of the laser beam path in the alignment method, within refraction field in the vicinity of a tunnel kiln
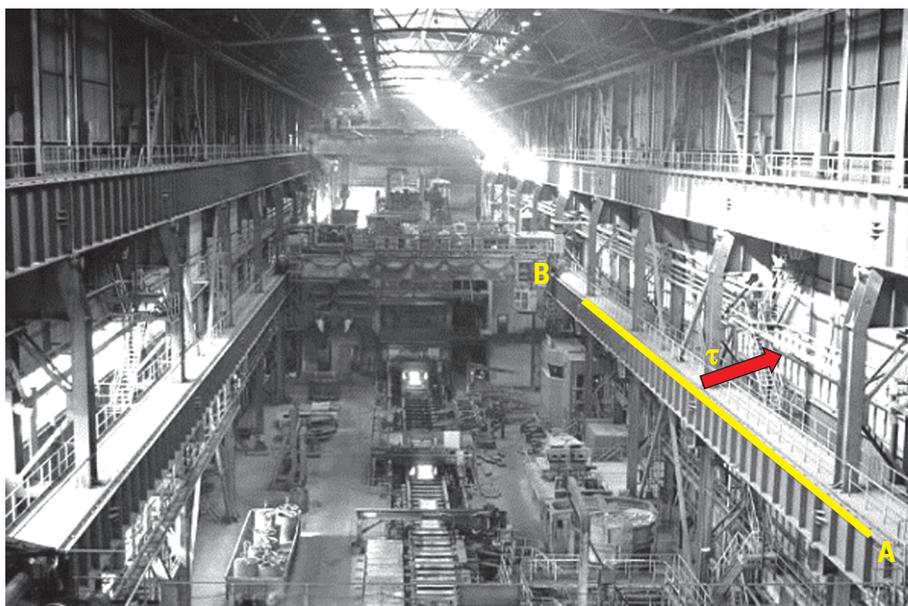
Photo by authors

Fig. 3. Production hall in the rolling mill – a hot strip mill with kilns emitting horizontal gradients of temperature (up to approx. $4\ \mathrm{K \cdot m^{-1}}$ at the distance of 3 m from the kiln batteries), in the direct vicinity of the A–B line base (Determining major shifts of horizontal control points on I-beam girders)

When measuring meteorological parameters describing atmospheric and micro-climatic conditions in the given setting/object (including temperature, temperature gradient, atmospheric pressure, humidity, smoke opacity etc.), assuming their slight variations in time and space at measurement points, line effect of refraction can be calculated using the formulae given in the publications [Moritz 1962, Bryś 1995, 1996 and Hennes 1998]. Taking into account specific conditions in the enclosed space, where straightness measurements are envisaged using a laser measuring set, the following assumptions should be made as concerns the spatial refraction curve:

- the curve has a constant curvature in the discussed refraction field,
- refraction curve on the object constitutes a section of a cylinder,
- there is no air circulation in the production hall,
- in the established measurement sections, that is in the points of alignmetric line of "i", gradients of temperatures τ_i are determined.

In Figure 2, schematic design assumptions of the studied object were shown, in the environment of the refraction field, as well as linear refraction effects occurring for each control point.

3. Model design assumptions for the impact of horizontal reffraction effect on the path of laser beam

Precise formula which – with the above assumptions – facilitates the calculation of line corrections of q_i for the impact of horizontal refraction, was derived based on the mathematical elaborations presented in [Holejko 1987 and Bryś 1994, 1995, 1996].

The modified and final formula, therefore, is as follows:

$$q_i = -0.405 \cdot 10^{-4} \cdot S_i (S - S_i) \cdot \frac{P_{śr}}{T_{śr}^2} \cdot dT_{śr} \tag{4}$$

where:

S_i – distance between the point of measurement (control point), and the laser's position [m],

S – distance between the target point, and the laser's position [m],

$T_{śr}$ – average air temperature in the hall K,

$P_{śr}$ – average atmospheric pressure [hPa],

τ_{X_i} – horizontal gradient of air temperature per 1 meter ($\Delta T / (1m)$).

$dT_{śr} = T_{śr2} - T_{śr1}$ (difference between average values of particular temperatures T_1 and T_2 measured in the measurement points at 1 m distance, for the given measurement section). Because the refraction angles (horizontal and vertical) are covariant, the formula for the vertical refraction angle in the vertical plane is obtained through a 100[g] rotation of the coordinate system about the Y axis.

When conducting alignmetric measurements in the space of the refractive field, the following meteorological parameters are calculated for the studied engineering object:

- temperature (T) measured with a laboratory thermometer with a maximum calculation error of $\sigma_T = 0.1$ K,
- atmospheric pressure (P) measured with an electronic barometer with a maximum calculation error of $\sigma_P = 0.1$ hPa,
- the difference of temperatures dT measured with an electronic meter of difference in air temperatures, with a maximum calculation error of $\sigma_{\Delta T} = 0.02$ K · m⁻¹.

As an example, the electronic meter can be constructed using two interconnected sensors of mercury vapour thermocouples [Bryś 1994]. The value of calculated difference of temperatures $dT = T_2 – T_1$ is read on the micro-voltometer scale calibrated in °C (Centigrade).

4. Assumptions for measurements and design

For the given range values of the studied elongated engineering object, deflection of the path of the laser beam can be calculated as a line impact of the horizontal refraction q_i of selected measurement points, assuming the following data:

S	–	143.488 m (length of the alignmetric base),
$S – S_i$	–	distance to the control point,
$P_{śr}$	–	1020 hPa,
$T_{śr}$	–	313 K,
dT_i	–	horizontal temperature gradient (calculated in K · m⁻¹) at the control point.

The assumed output data and the data calculated for the practical measurement tasks were shown for comparison in Table 1.

Table 1. Comparison of sample data – assumed and calculated – for the determination of the impact of horizontal alignment refraction on the path of laser beam indoors (in a closed space); for the calculation of the value of line refraction effect – adjusted by q_i according to Figure 2

No. of control point	S [m]	S_i [m]	$S – S_i$ [m]	$\tau_{X_i} = \Delta T/1m$ [K · 1 m⁻¹]	q_i [mm] Refraction adjustment
a	b	c	d	e	f
6	143.448	73.448	70	−1.82	**−4.0**
5	143.448	63.448	80	−1.15	**−2.5**
4	143.448	53.448	90	−0.94	**−1.9**
3	143.448	43.448	100	−0.87	**−1.6**
2	143.448	33.448	110	−0.78	**−1.2**
1	143.448	23.448	120	−0.21	**−0.3**

Source: authors' study

5. Conclusions

Based on the measurements conducted during the experiment, the values of vectors of the horizontal shifts of the steel bar edges in the vicinity of the kilns in the steel mill (temperature of ca. 1200 degrees Centigrade), significantly exceed industry requirements in terms of precision. Presented herein, the model formulae for refraction adjustments in alignment refractions models of the highest precision [Holejko 1987, Bryś 1994] with standard deviation of 0.1 mm can find application in the following geodesic engineering tasks:

· controlling, aligning, adjusting and setting the elements of axes of machines and technical appliances in the energy industry,

· determining space coordinates of elements of turbines, rotating kilns, axes of transmission shafts, etc.

· spatial control of the geometry of production lines in continuous steel casting, conveyor belts, production lines of cars and other machinery,

· calculating deformations and distortions of the concrete dam corpus,

· determining the geometry of the axis of crane rail tracks, retaining walls, etc.

Degree of precision postulated for indoor measurements, that is $\sigma_{X, Y, Z} = |0.05 - 0.10|$ mm, can be achieved only when the model adjustments (corrections) will be applied to the obtained geodesic measurements done using the alignment method. As required in the industry instructions, the precision of calculating vectors of the horizontal shifts requires the measurement of temperature gradients with calculation uncertainty of $\sigma_{\Delta T} = |0.02 \text{ K} \cdot \text{m}^{-1}|$.

This degree of precision is provided by modern sensors and electronic temperature gradient meters [Werner 1985, Bryś 1994]. As demonstrated in the experimental measurements conducted in the steel mill, the line alignment refraction effects can be determined based on the formulae presented in this paper with the reliability of approx. 70–90%, assuming precise and current meteorological parameters, measured using paired and tested sensors, meters and other measurement devices [Holejko 1987]. We should stress, however, that mathematical-physical formulae are only that – formulae, and that they should be modified and adjusted to the current conditions of the refractive space of the given objects emitting temperature gradients [Bryś 1996].

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