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POSSIBLE TESTING OF GEODETIC EQUIPMENT USING THE METHODS OF MATHEMATICAL STATISTICS

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Summary

The work presents selected methods of mathematical statistics applied to the examination of the correctness of geodetic equipment operation.
Specifically, measurements made with electronic total stations will be tested using statistical compatibility tests and identity tests of empirical measurement error distributions with the theoretical normal distribution N (0;1).

Keywords

statistical compatibility tests • statistical identity tests • geodetic measurements • error distribution • testing of measurement correctness • statistical hypothesis

1. Introduction

Correctly made geodetic measurements should present a normal distribution of errors. If this requirement is met, then the Gaussian least-squares method can be used to adjust the measurements made – either by using an intermediary method or a contingent method.

In cases where measurement errors do not follow standard distribution, it is not possible to use the Gaussian methods for their mathematical interpretation [Babushka et al. 2014, Czaja 1996, Ney 1970].

Statistical tests of compliance or tests of identity can be used to verify whether the measurements follow normal distribution. [Ney 1970].

Compliance tests can be used, for instance:

- λ – Kolmogorov's
- X^2 – Pearson' s
- Shapiro–Wilk of distribution symmetry and flattering [Greń 1970, Krysicki et al. 1986]

Identity tests can be applied, such as:

- Kolmogorov–Smirnov test,

- Confidence intervals,
- Wilcoxon's rank-sum test [Kaczmarek 1970].

The present work has two objectives:

1. Testing the correctness of measurements conducted using electronic total stations: Leica TS02, serial number: 1313851 and Topcon 105N, serial number: 6H7039 in the possession of the Department of Geodesy at the University of Agriculture in Krakow.
2. Dissemination of statistical methods among surveyors, particularly, less frequently used tests:
   - compatibility:
     - the Anderson–Darling test,
     - the Watson test;
   - identity: T signed-rank test [Puchalski 1969].

2. Characteristics of compliance tests applied

1. Anderson–Darling test statistic is determined with the formula [Kasietczuk 1993]:

$$A^2 = n\int_{-\infty}^{+\infty} \frac{\left[Fn(x) - F(x)\right]^2}{F(x)\left(1 - F(x)\right)} dF(x)$$

where:
$Fn(x)$ – empirical distribution function,
$F(x)$ – normal distribution function,
n – statistical sample size.

For practical calculations, the following formula is used:

$$A^2 = \frac{-\left[\sum_{i=1}^{n}\left(2i-1\right)[\ln z_i + \ln(1 - z_{n+1-i})]\right]}{n} - n$$

and:

$$A_1^2 = \left(1 + \frac{4}{n} - \frac{25}{n^2}\right) \cdot A^2$$

where:
$z_i = F(u_i)$
$F(u)$ – distribution function N (0;1).

2. U_1^2 statistic of the Watson test is determined by the following formula [Kasietczuk 1993]:

$$U^2 = W^2 - n\left(\sum_{i=1}^{n}\frac{z_i}{n} - 0{,}5\right)^2$$

$$U_1^2 = \left(1 + \frac{0.5}{n}\right)\cdot U^2$$

where:

$$w^2 = \sum_{i=1}^{n}\left[z_i - \frac{2i-1}{2n}\right]^2 + \frac{1}{12^n}$$

$z_i = F(u_i)$,

$F(u)$ – distribution function $N(0;1)$,

$u_i = \frac{x_i - \bar{x}}{\partial}$ – statistical sample size.

3. Testing the correctness of measurements conducted using the Leica TS02 total station (as illustrated with the example of length measurements)

In order to verify the correctness of measurements, the side length measurement was carried out in four series of 9 measurements in each series.

The series of measurements is understood as a set of activities and readings necessary to determine the length of a single side of the matrix. In this way, 36 length measurement results were obtained.

The results of measurements and calculations are presented in Table 1.

These measurements were tested using statistical compliance tests:

- The Anderson – Darling test,
- The Watson test.

These studies were meant to answer the question whether the distribution of measurement errors is consistent with the theoretical normal distribution, because only in this case the Gaussian least-squares method can be applied for their alignment [Czaja 1996, Siejka 2017].

Table 1. The Anderson–Darling test

No.	x_i [m]	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	u_i	$z_i = F(u_i)$	z_{n+1+i}	$\ln z_i + \ln(1 - z_{n+1-1})$	$(2_i - 1)\cdot \ln z_i + \ln(1 - z_{n+1-1})$
1	213.543	–0.00192	0.00000367	–1.479	0.0694	0.9452	–5.5719	–5.5719
2	213.543	–0.00192	0.00000367	–1.479	0.0694	0.9452	–5.5719	–16.7157
3	213.543	–0.00192	0.00000367	–1.479	0.0694	0.9452	–5.5719	–27.8595

Table 1. cont.

No.	x_i [m]	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	u_i	$z_i = F(u_i)$	z_{n+1+i}	$\ln z_i + \ln(1 - z_{n+1-1})$	$(2_i - 1) \cdot \ln z_i + \ln(1 - z_{n+1-1})$
4	543	–0.00192	0.00000367	–1.479	0.0694	0.9452	–5.5719	–39.0033
5	543	–0.00192	0.00000367	–1.479	0.0694	0.9452	–5.5719	–50.1471
6	543	–0.00192	0.00000367	–1.479	0.0694	0.7967	–4.2609	–55.3917
7	544	–0.00092	0.00000084	–0.707	0.2389	0.7967	–3.0248	–39.3224
8	544	–0.00092	0.00000084	–0.707	0.2389	0.7967	–3.0248	–45.3720
9	544	–0.00092	0.00000084	–0.707	0.2389	0.7967	–3.0248	–51.4216
10	544	–0.00092	0.00000084	–0.707	0.2389	0.7967	–3.0248	–57.4712
11	544	–0.00092	0.00000084	–0.707	0.2389	0.7967	–3.0248	–63.5208
12	544	–0.00092	0.00000084	–0.707	0.2389	0.7967	–3.0248	–69.5704
13	544	–0.00092	0.00000084	–0.707	0.2389	0.5239	–2.1738	–53.3450
14	544	–0.00092	0.00000084	–0.707	0.2389	0.5239	–2.1738	–58.6926
15	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–40.2694
16	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–43.0466
17	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–45.8238
18	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–48.6010
19	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–51.3782
20	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–54.1554
21	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–56.9326
22	545	0.00008	0.00000001	0.064	0.5239	0.5239	–1.3886	–59.7098
23	545	0.00008	0.00000001	0.064	0.5239	0.2389	–0.9194	–41.3730
24	545	0.00008	0.00000001	0.064	0.5239	0.2389	–0.9194	–43.2118
25	546	0.00108	0.00000117	0.836	0.7967	0.2389	–0.5003	–24.5147
26	546	0.00108	0.00000117	0.836	0.7967	0.2389	–0.5003	–25.5153
27	546	0.00108	0.00000117	0.836	0.7967	0.2389	–0.5003	–26.5159
28	546	0.00108	0.00000117	0.836	0.7967	0.2389	–0.5003	–27.5165
29	546	0.00108	0.00000117	0.836	0.7967	0.2389	–0.5003	–28.5171
30	546	0.00108	0.00000117	0.836	0.7967	0.2389	–0.5003	–29.5177
31	546	0.00108	0.00000117	0.836	0.7967	0.0694	–0.2992	–18.2512
32	547	0.00208	0.00000434	1.608	0.9452	0.0694	–0.1283	–8.0829

33	547	0.00208	0.00000434	1.608	0.9452	0.0694	−0.1283	−8.3395
34	547	0.00208	0.00000434	1.608	0.9452	0.0694	−0.1283	−8.5961
35	547	0.00208	0.00000434	1.608	0.9452	0.0694	−0.1283	−8.8527
36	547	0.00208	0.00000434	1.608	0.9452	0.0694	−0.1283	−9.1093
Σ	7687.617	−0.0001	0.00005875	−0.093				−1342.2357

$$\overline{x} = 213.5449 \text{ m}; \; V(x) = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = 0.000001679; \; \delta = 0.001296$$

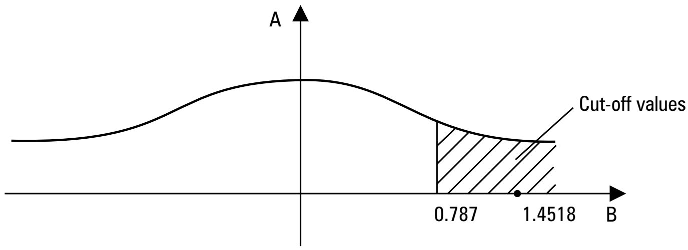
$$u = \frac{x_i - \overline{x}}{\delta}$$

$$A^2 = 1.2843; \; A_1^2 = 1.1304 \cdot 1.2843 \cong 1.4518$$

$$A_{1kryt}^2 = \text{for } \alpha = 0.05 \text{ is } 0.787$$

$$1.4518 > 0.787$$

The A–D test is skewed right.



At the significance level of $\alpha = 0.05$, the hypothesis about the conformity of the empirical distribution with the distribution of N (0;1) is rejected: the measurements contain systematic errors.

Table 2. The Watson test

No.	x_i [m]	$z_i = F(u_i)$	$\frac{2i-1}{2n}$	$z_i - \frac{2i-1}{2n}$	$\left[z_i - \frac{2i-1}{2n}\right]^2$
1	213.543	0.0694	0.0139	0.0555	0.0031
2	213.543	0.0694	0.0417	0.0277	0.0008
3	213.543	0.0694	0.0694	0.0000	0.0000
4	543	0.0694	0.0972	−0.0278	0.0008

Table 2. cont.

No.	x_i [m]	$z_i = F(u_i)$	$\frac{2i-1}{2n}$	$z_i - \frac{2i-1}{2n}$	$\left[z_i - \frac{2i-1}{2n}\right]^2$
5	543	0.0694	0.1250	–0.0556	0.0031
6	543	0.0694	0.1528	–0.0834	0.0070
7	544	0.2389	0.1806	0.0583	0.0034
8	544	0.2389	0.2083	0.0306	0.0009
9	544	0.2389	0.2361	0.0028	0.0000
10	544	0.2389	0.2639	–0.0250	0.0006
11	544	0.2389	0.2917	–0.0528	0.0028
12	544	0.2389	0.3194	–0.0805	0.0065
13	544	0.2389	0.3472	–0.1083	0.0117
14	544	0.2389	0.3750	–0.1361	0.0185
15	545	0.5239	0.4028	0.1211	0.0147
16	545	0.5239	0.4306	0.0933	0.0087
17	545	0.5239	0.4583	0.0656	0.0043
18	545	0.5239	0.4861	0.0378	0.0014
19	545	0.5239	0.5139	0.0100	0.0001
20	545	0.5239	0.5417	–0.0178	0.0003
21	545	0.5239	0.5694	–0.0455	0.0021
22	545	0.5239	0.5972	–0.0733	0.0054
23	545	0.5239	0.6250	–0.1011	0.0102
24	545	0.5239	0.6528	–0.1289	0.0166
25	546	0.7967	0.6806	0.1161	0.0135
26	546	0.7967	0.7083	0.0884	0.0078
27	546	0.7967	0.7361	0.0606	0.0037
28	546	0.7967	0.7639	0.0328	0.0011
29	546	0.7967	0.7917	0.0050	0.0000
30	546	0.7967	0.8194	–0.0227	0.0005
31	546	0.7967	0.8472	–0.0505	0.0026
32	547	0.9452	0.8750	0.0702	0.0049

33	547	0.9453	0.9028	0.0425	0.0018
34	547	0.9454	0.9306	0.0148	0.0002
35	547	0.9455	0.9583	–0.0128	0.0002
36	547	0.9456	0.9861	–0.0405	0.0016
Σ	7687.617				0.1609

$$\overline{x} = 213.5449 \text{ m}$$

$$u = \frac{x_i - \overline{x}}{\delta};\ W^2 = \sum_{i=1}^{n}\left[z_i - \frac{2i-1}{2n}\right]^2 + \frac{1}{12n}$$

$$W^2 = 0.1448 + 0.0023 = 0.1471$$

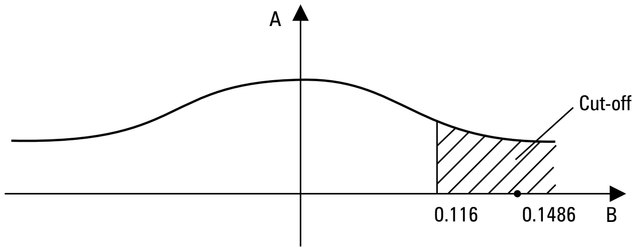
$$u^2 = w^2 - n\left(\frac{\sum z_i}{n} - 0.5\right)^2 = 0.1471 - 36 \cdot (0.4964 - 0.5)^2 = 0.1466$$

$$u_1^2 = \left(1 + \frac{0.5}{n}\right) \div u^2 = 1.0139 \cdot 0.1466 = 0.1486$$

$$u_{1\text{kryt.}}^2 = 0.116 : \text{from Watson's tables for } \alpha = 0.05$$

$$0.1486 > 0.116$$

The W test is skewed right:



At the significance level of $\alpha = 0.05$, the H_0 hypothesis should be rejected: the measurements are subject to systematic errors. This is confirmed by the Anderson–Darling test result.

4. Testing the correctness of measurements made using the Leica TS02 total station (as illustrated with the example of angular measurements)

In order to test whether the tested Leica TS02 total station is working correctly, a 9-fold measurement of three angles in a triangle was conducted, resulting in 9 deviations of the triangles' "closures".

These angles were also measured using the Topcon 105 N instrument, which had been checked beforehand, and the distribution of its measurement errors proved to be consistent with the normal distribution.

Using the Topcon total station, the angles were measured 12 times. Each measurement was made in two series, in 2 positions of the telescope.

The value of the triangles' closures in gradian seconds for both total stations are listed in the table 3.

Table 3. The value of the triangles' closures in gradian seconds

No.	Leica TS02 (sample no. 1)	Topcon 105 N (sample no. 2)
1	–2	–2
2	–1	2
3	5	2
4	2	0
5	0	–3
6	–4	–1
7	5	0
8	4	2
9	2	3
10		–3
11		–1
12		1

Verifying the correctness of angular measurements by statistical sampling test – T signed-rank test – consists in comparing the results of measurements made with the tested instrument (in this case, Leica TS02) with the results of measurements made with the previously tested, properly functioning instrument (in this case, Topcno 15N) [Piasek 2000, Siejka 2017].

As a result of fieldwork, two random samples with numbers were obtained:

- Sample 1 (Leica): $n_1 = 9$
- Sample 2 (Topcon): $n_2 = 12$

All the observations of both samples are ranked according to increasing values, and they are sequentially numbered. Then, the sum of the rank for each sample is calculated separately. The sum of consecutive numbers for the smaller sample is T_m, and for the larger sample, it is T_w.

The zero hypothesis (H_0) that we are thus verifying is the assumption that there is no difference between the distributions of both samples – therefore, that the ratio of T_m rank sum to the T_w rank sum is the same as the ratio of the number of elements of the first sample n_1 to the number of elements of the second sample n_2, that is:

$$\frac{T_m}{T_w} \cong \frac{n_1}{n_2}$$

Not rejecting the H_0 hypothesis would inevitably lead to the conclusion about the normality of the error distribution of the tested instrument.

In order to verify H_0, the value of T_m is compared with the value of $T_{0.05}$ taken from the *T* test tables.

In the case where $T_{0.05} < T_m$ – there are no grounds to reject H_0, that is, the results of the samples derive from the general population with the same distribution.

If the results of both measurements are arranged in one series according to ascending values, we get (Table 4).

Table 4. The series of ascending values

Rank sum	Deviation [cc]	No. of the sample
1	–4	I
2	–4	I
3	–3	II
4	–3	II
5,5	–2	II
5,5	–2	I
8	–1	I
8	–1	II
8	–1	II
11	0	II
11	0	II
11	0	I
13	1	II
16	2	II
16	2	II
16	2	II
16	2	I
16	2	I
19	3	II
20	5	I
21	5	I

The deviation of –4 occurs in two cases, but since both derive from the same series of measurements, they are numbered consecutively (No. 1 and No. 2).

Similarly, deviation –3 (No. 3 and No. 4).

The deviation of –2 occurs twice, but in two different series. These two results should be assigned numbers 5 and 6, but in this case, one common number for the same results is calculated:

$$\frac{5+6}{2} = 5.5$$

Similarly, for deviation –1:

$$\frac{7+8+9}{3} = 8 \text{ and so forth.}$$

Thus we arrive at: $T_m = 1 + 2 + 5.5 + 8 + 11 + 16 + 16 + 20 + 21 = 100.5$

$T_w = 3 + 4 + 5.5 + 8 + 8 + 11 + 11 + 13 + 16 + 16 + 16 + 19 = 130.5$

$$T = T_m + T_w = 231;\quad S = \frac{n \cdot (n+1)}{2} = \frac{21 \cdot 22}{2} = 231$$

Which confirms the calculations to be correct.

It follows from the calculations that: $\frac{n_1}{n_2} = \frac{9}{12} = 0.75$

whereas $\frac{T_m}{T_w} = \frac{100.5}{130.5} \cong 0.77$ which confirms the required assumptions.

$T_m = 100.5$, whereas, from the test tables, $T - T_{0.05}$ for $n_1 = 9$ and $n_2 = 12$ amounts to 71, therefore:

$$T_{0.05} = 71 < 100.5 = T_m$$

This indicates that there are no grounds to reject H_0 at the significance level of $\alpha = 0.05$, that is, that the angular measurements made with the tested total station are correct.

5. Conclusions

1. The study to verify the correctness of length measurements performed with the Leica TS02 total station shows that according to the Anderson–Darling test these measurements contain systematic errors. The value of the *A*12 statistic of this test remains within the cut-off range, because:

$$A_1^2 = 1.4518 > 0.787 = A_{1kryt}^2$$

2. The Watson test confirmed the previous test result, because in this case the value of the *u*12 statistic remains within the cut-off range of the test:

$$u_1^2 = 0.1486 > 0.116 = u_{1kryt}^2$$

3. However, angular measurements carried out with the use of the *T* signed-rank test show that the angular measurements made with the Leica TS02 total station do not

contain systematic errors, and that their distribution of errors corresponds to the theoretical normal distribution. Test statistic T:

$$T_{0.05} = 71 < 100.5 = T_m$$

4. Previous tests indicate another general conclusion, namely that before the measurement works, the tested total station should be sent for rectification.
5. The whole study also shows that statistical compliance tests and identity tests should most definitely be used for this kind of research. By working in this way, it is possible to avoid the wrong decision, and instead choose the correct method for aligning geodetic observations.

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