

POINT POSITION ACCURACY IN A VECTOR GNSS NETWORK AND THE WAY IT IS LINKED TO REFERENCE STATIONS

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Summary

The aim of this study is to assess the impact of the location (distribution) of reference points (reference stations of the ASG-EUPOS system) on the accuracy of the final determination of the local measurement grid points. The research was carried out in terms of the possibility of using the static GNSS method to determine displacements, both relative (vector lengths) and absolute (coordinates in the spatial system). A mathematical record of the computational process (functional model and stochastic model) was presented, on the basis of which the test vector network was adjusted (indirect method) and the accuracy assessment after the adjustment was performed. The subject of the numerical tests were the actual measurement results of a part of the geodetic network (GNSS vectors) established in the mining area (the results of one of the periodic measurement cycles were used). Numerical analyses take into account several different variants of establishing the network: depending on the location (direction east–west, north–south) and the number of ASG-EUPOS stations used. The following parameters (relating to the designated positions) were adopted as comparative criteria: coordinate deviation (in the Cartesian geocentric system) from the reference values, spatial length deviation between the designated points from its reference value, mean coordinate errors, error in the position of a point in three-dimensional space, length mean error as a function of adjusted observations (using the law of transfer of errors of mean correlated quantities). Particular attention was paid to the discrepancy between the adjustment results for different systems of reference to the ASG-EUPOS stations. On the basis of the performed calculation tests and the performed comparative analyses, conclusions were compiled that may be helpful in planning periodic measurements for the purpose of determining land displacements.

Keywords

GNSS vector network • establishing a geodetic control network • ASG-EUPOS reference station • rigorous adjustment • determining terrain displacements

1. Introduction

The technique of GNSS satellite measurements gives the opportunity to easily connect the measurement object with the geodetic network, thanks to the possibility of using vectors of any length. This is important, for example, when determining absolute

displacements in areas prone to displacement [Góral and Szewczyk 2004, Baryła et al. 2007, Wiśniewski and Kamiński 2020]. Such possibilities of establishing the network are provided by the ASG-EUPOS system [Bosy et al. 2008], whose main function is to ensure a uniform (nationwide) system of spatial references. According to the studies [GUGIK 2011, 2013], the accuracy of determining the coordinates when using computing services (e.g. POZGEO [Pażus 2009]) available in this system is 0.01 m for the static method, however, the actual accuracy also depends on the type of measuring equipment, observation time, post-processing software, etc. – the actual error in determining the position may range from a few to over a dozen centimetres. The research conducted by the authors of the study [Dawidowicz et al. 2007] shows, however, that determining a point height with an accuracy of 1–2 cm with long vectors (of ca. 20 km) requires long observation sessions (over two hours), which in turn is associated with a greater cost of measurement. Manufacturers of GNSS receivers [Leica 2009] adopt the following principle of determining the accuracy of the position using the static method: 5–10 mm + 0.5 ppm (for example, for a vector of 50 km it gives approx. ± 3 cm). The author of another paper [Kadaj 2007] claims, however, that long GNSS vectors are characterised by constant (sub-centimetre) accuracy and therefore they constitute a simple method of improving the structure of classical networks in terms of absolute accuracy and reliability. A fundamental question arises here: does the GNSS measurement technology (using the ASG-EUPOS system) really lead to a qualitative improvement of the entire observation system in any situation? For displacement measurements, precise situational and height measurements are required (using total stations), while in order to determine settlements, geometric levelling (precise or technical) with mean errors in the height difference measurement below 0.1 mm [Bałut 1997, Prószyński and Kwaśniak 2006].

As it is known, long GNSS vectors used with reference to reference stations (in the order of several dozen kilometres) are burdened with significant errors, both in terms of the vector length and its orientation in space [Kadaj 2007]. Measurement errors, in turn, decide on the accuracy of position determination (mean error of the point position) and indirectly affect the precision of the values determined as a function of the coordinates of the points (e.g. the distance between the points). The obtained accuracy of the determinations may, consequently, determine the necessity to choose a different measurement method that meets the assumed requirements, even in spite of the greater costs and labour-intensity of field work.

As part of the research carried out in the course of the work [Gargula 2011], a series of numerical tests were carried out in terms of determining displacements (on two test objects), and the analysis of the obtained results allowed, inter alia, to determine a certain relationship between the conditions of referencing the ASG-EUPOS network points (e.g. the number of points or the directions of their location in relation to the object) and the parameters describing the position (relative and absolute) of the points of the tested control network. This paper is an attempt to continue this research. Similar research issues were also undertaken in other studies, e.g. [Gargula 2009, 2010, 2011a, 2019].

2. Description of the research performed

The test object was a part of the control network (Fig. 1a: side 4–5), measured with reference to the reference points *KATO*, *KRAW*, *ZYWI* and *WODZ* (of the ASG-EUPOS system). The vectors between the studied points (4–5) and the reference vectors (to the reference points) were measured using the GNSS static method (2 receivers, measurement time: 60 minutes). The control network was established to study the displacements of the area in the mining area (close to Chełm Śląski, near Katowice). Figure 1 shows several research variants (V_1÷V_6), differing in the way the test section is linked to the reference points:

- Option V_1: reference to 4 points in E, W, N, S directions (E – east, W – west, N – north, S – south);
- Variant V_2: reference to 3 points in W, N, S directions;
- Variant V_3: reference to 2 points in N, S directions;
- Variant V_4: reference to 2 points in W, N directions;
- Variant V_5: reference to 1 point in the N direction;
- Variant V_6: Reference to 2 points in E, W directions.

For the purposes of this study, the results of measurements (Table 1), necessary for the implementation of variant V_1 (Fig. 1a), obtained on the basis of postprocessing (GNSS vectors and their mean errors) were used. To complete the set of output data, the coordinates of the reference stations and the approximate coordinates of the designated points (necessary to adjust the observations using the indirect method, according to the least squares procedure) were also compiled (in Table 2).

Table 1. List of observations to be adjusted and their mean errors

Vector markings		Observations (components of GNSS vectors) [m]			Mean errors of observations [m]		
From	To	ΔX	ΔY	ΔZ	$m_{\Delta X}$	$m_{\Delta Y}$	$m_{\Delta Z}$
5	4	-1.5898	23.8237	-5.1855	0.0066	0.0051	0.0052
KATO	4	8881.6840	13105.5420	-10642.9751	0.0055	0.0043	0.0047
KATO	5	8883.2690	13081.7339	-10637.7916	0.0166	0.0131	0.0143
KRAW	4	14937.9164	-51822.2599	2743.0312	0.0064	0.0053	0.0060
KRAW	5	14939.4976	-51846.0695	2748.2310	0.0094	0.0080	0.0085
WODZ	4	-24824.6801	45254.5225	7433.1278	0.0062	0.0051	0.0057
WODZ	5	-24823.1195	45230.6841	7438.3197	0.0158	0.0139	0.0138
ZYWI	4	-32759.2432	-14263.6766	29831.6776	0.0104	0.0073	0.0085
ZYWI	5	-32757.6232	-14287.4916	29836.8737	0.0106	0.0063	0.0072

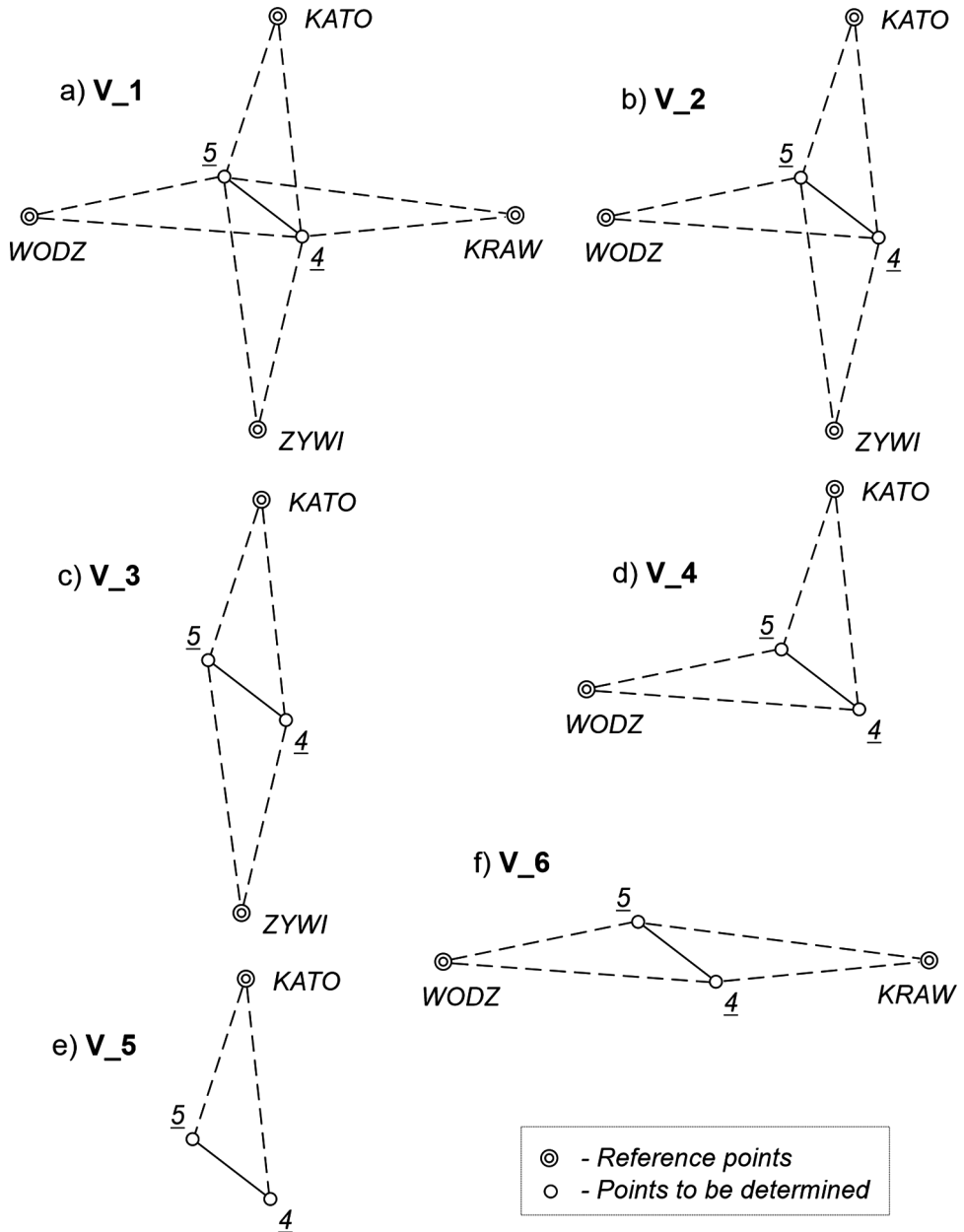


Fig. 1. Research variants (a-f)

Table 2. Coordinates of reference stations (*KATO*, *KRAW*, *WODZ*, *ZYWI*) and approximate coordinates (*) of the points to be determined (4, 5) – ETRF’89 geocentric system

Point	X [m]	Y [m]	Z [m]
KATO	3862992.3806	1332822.6741	4881105.4573
KRAW	3856936.1743	1397750.4815	4867719.4488
WODZ	3896698.7807	1300673.7066	4863029.3737
ZYWI	3904633.3207	1360191.8920	4840630.7894
4*	3871874.0646	1345928.2161	4870462.4822
5*	3871875.6496	1345904.4080	4870467.6657

The aim of each of the research variants (Fig. 1) was to adjust the given observation system and determine the coordinates of the points 4, 5. The adjustment results also made it possible to calculate the spatial length of the side 4-5 (used as a comparative parameter when monitoring the relative displacements in successive cycles of periodic measurements) and to analyse the accuracy of the determined values (mean errors of coordinates and length).

The mathematical record of the adjustment process (functional model) and accuracy assessment (stochastic model) will be presented below.

The adjustment of the vector GNSS network according to the intermediary method begins with the compilation of the observational equations for the three components of each *j*-*k* vector:

$$\begin{cases} \Delta x_{jk} + v_{jk}^{(\Delta x)} = x_k - x_j \\ \Delta y_{jk} + v_{jk}^{(\Delta y)} = y_k - y_j \\ \Delta z_{jk} + v_{jk}^{(\Delta z)} = z_k - z_j \end{cases} \quad (1)$$

$$\begin{cases} v_{jk}^{(\Delta x)} = \delta x_k - \delta x_j + l_{jk}^{(\Delta x)} \\ v_{jk}^{(\Delta y)} = \delta y_k - \delta y_j + l_{jk}^{(\Delta y)} \\ v_{jk}^{(\Delta z)} = \delta z_k - \delta z_j + l_{jk}^{(\Delta z)} \end{cases} \quad (2)$$

$$\begin{cases} l_{jk}^{(\Delta x)} = \Delta x_{jk}^{(0)} - \Delta x_{jk} \\ l_{jk}^{(\Delta y)} = \Delta y_{jk}^{(0)} - \Delta y_{jk} \\ l_{jk}^{(\Delta z)} = \Delta z_{jk}^{(0)} - \Delta z_{jk} \end{cases} \quad (3)$$

$$\begin{cases} \Delta x_{jk}^{(0)} = x_k^{(0)} - x_j^{(0)} \\ \Delta y_{jk}^{(0)} = y_k^{(0)} - y_j^{(0)} \\ \Delta z_{jk}^{(0)} = z_k^{(0)} - z_j^{(0)} \end{cases} \quad (4)$$

where:

- $l^{(\Delta x)}, l^{(\Delta y)}, l^{(\Delta z)}$ – absolute terms in correction equations (2),
- $v^{(\Delta x)}, v^{(\Delta y)}, v^{(\Delta z)}$ – searched corrections to component elements of the GNSS vector,
- $\Delta x^{(0)}, \Delta y^{(0)}, \Delta z^{(0)}$ – approximate values of the vector components,
- $\delta x, \delta y, \delta z$ – searched increments (corrections) to approximate coordinates,
- $x^{(0)}, y^{(0)}, z^{(0)}$ – approximate coordinates.

The system of correction equations (2) is written in the matrix form:

$$\mathbf{V} = \mathbf{A} \cdot \mathbf{X} - \mathbf{L} \quad (5)$$

where:

$$\mathbf{V} = \left[\left\{ v_{jk}^{(\Delta x)}; v_{jk}^{(\Delta y)}; v_{jk}^{(\Delta z)} \right\} \right]^T \quad - \text{vector of determined corrections,}$$

\mathbf{A} – matrix of coefficients with unknowns, in this case these coefficients are: 1 (for the end point k) or -1 (for the starting point j),

$$\mathbf{X} = \left[\left\{ \delta x_j; \delta y_j; \delta z_j; \delta x_k; \delta y_k; \delta z_k \right\} \right]^T \quad - \text{vector of unknowns, i.e. increments to approximate coordinates,}$$

$$\mathbf{L} = \left[\left\{ l_{jk}^{(\Delta x)}; l_{jk}^{(\Delta y)}; l_{jk}^{(\Delta z)} \right\} \right]^T \quad - \text{vector of absolute terms (11).}$$

The estimated vector of unknowns $\hat{\mathbf{X}}$ is calculated using the method known from the adjustment calculus [Wiśniewski 2005], resulting from the imposition of the least squares condition ($\mathbf{V}^T \cdot \mathbf{P} \cdot \mathbf{V} = \mathbf{min.}$) on the system (5):

$$\hat{\mathbf{X}} = (\mathbf{A}^T \cdot \mathbf{P} \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \mathbf{P} \cdot \mathbf{L} \quad (6)$$

where \mathbf{P} is the weight (diagonal) matrix, compiled on the basis of errors of mean a priori of vector components ($m_{jk}^{(\Delta x)}, m_{jk}^{(\Delta y)}, m_{jk}^{(\Delta z)}$):

$$\mathbf{P} = \text{diag} \left\{ \frac{1}{\left(m_{jk}^{(\Delta x)} \right)^2}; \frac{1}{\left(m_{jk}^{(\Delta y)} \right)^2}; \frac{1}{\left(m_{jk}^{(\Delta z)} \right)^2} \right\} \quad (7)$$

The next step is to substitute the vector $\hat{\mathbf{X}}$ (6) for the vector of unknowns \mathbf{X} (5) and calculate the vector of observational corrections \mathbf{V} , which, in turn, provide the basis for adjusting the observations – the left side of the observation equations (1).

Information about errors in the means of the adjusted coordinates (m_x, m_y, m_z) can be found on the diagonal of the covariance matrix of the vector \mathbf{X} :

$$\mathbf{Q}_x = m_0 \cdot (\mathbf{A}^T \cdot \mathbf{P} \cdot \mathbf{A})^{-1} \rightarrow \text{diag} \{ \mathbf{Q}_x \} = \{ m_x; m_y; m_z \} \tag{8}$$

$$m_0 = \sqrt{\frac{\mathbf{V}^T \cdot \mathbf{P} \cdot \mathbf{V}}{r}} \tag{9}$$

where:

- m_0 – mean unit error of observations,
- r – the number of oversized observations.

Then, for each designated point, a single parameter characterising its accuracy will be calculated – the error of the point location in the spatial Cartesian system:

$$m_p = \sqrt{m_x^2 + m_y^2 + m_z^2} \tag{10}$$

Since we are also interested in the relative (mutual) position of the points to be determined, then on the basis of adjusted observations ($\overline{\Delta x}, \overline{\Delta y}, \overline{\Delta z}$) we calculate the spatial distance (generally: for points $j-k$, in our example: 4-5):

$$d_{jk} = \sqrt{(\overline{\Delta x}_{jk})^2 + (\overline{\Delta y}_{jk})^2 + (\overline{\Delta z}_{jk})^2} \tag{11}$$

To estimate the accuracy of determined distance $j-k$ the law of transfer of mean errors for correlated quantities [Wiśniewski 2005] is applied:

$$m_{jk}^{(d)} = \sqrt{\left(\frac{\partial d_{jk}}{\partial (\Delta x_{jk})} \right)^2 \cdot (m_{jk}^{(\Delta x)})^2 + \left(\frac{\partial d_{jk}}{\partial (\Delta y_{jk})} \right)^2 \cdot (m_{jk}^{(\Delta y)})^2 + \left(\frac{\partial d_{jk}}{\partial (\Delta z_{jk})} \right)^2 \cdot (m_{jk}^{(\Delta z)})^2} \tag{12}$$

After calculating the partial derivatives (∂ symbol) with respect to the individual variables (jk vector components), we obtain the final formula:

$$m_{jk}^{(d)} = \frac{\sqrt{(\Delta x_{jk})^2 \cdot (m_{jk}^{(\Delta x)})^2 + (\Delta y_{jk})^2 \cdot (m_{jk}^{(\Delta y)})^2 + (\Delta z_{jk})^2 \cdot (m_{jk}^{(\Delta z)})^2}}{d_{jk}} \tag{13}$$

The calculation procedure written with the formulas (1)÷(13) was carried out (for the planned test variants) with the use of formulas developed in MS Excel.

3. Analysis of the results

As a result of the adjustment of the test networks in individual variants, the coordinates of the control network points and the spatial distance were obtained (Table 3 – for the purposes of comparative analysis, these data were recorded with a precision of 0.0001 m). Coordinate and length deviations (in relation to the initial version V_1, where the highest number of reference points occur) are summarised in Table 4 and illustrated in graphs (Fig. 2 and 3).

Table 3. Adjustment results (geocentric coordinates and spatial length)

Version	Point: 4			Point: 5			Side: 4-5
	X [m]	Y [m]	Z [m]	X [m]	Y [m]	Z [m]	d [m]
V_1	3871874.0832	1345928.2216	4870462.4852	3871875.6745	1345904.4013	4870467.6716	24.4303
V_2	3871874.0811	1345928.2210	4870462.4860	3871875.6739	1345904.3987	4870467.6704	24.4318
V_3	3871874.0717	1345928.2178	4870462.4785	3871875.6693	1345904.3975	4870467.6639	24.4304
V_4	3871874.0792	1345928.2217	4870462.4904	3871875.6657	1345904.3984	4870467.6768	24.4329
V_5	3871874.0642	1345928.2174	4870462.4820	3871875.6534	1345904.3956	4870467.6673	24.4314
V_6	3871874.0933	1345928.2263	4870462.4925	3871875.6775	1345904.4040	4870467.6799	24.4319

Table 4. Coordinate and side length deviations 4-5 (in relation to version V_1)

Deviations for version:	Point: 4			Point: 5			Side: 4-5
	f_x [mm]	f_y [mm]	f_z [mm]	f_x [mm]	f_y [mm]	f_z [mm]	f_d [mm]
V_2	-2.1	-0.7	0.8	-0.5	-2.6	-1.3	1.5
V_3	-11.5	-3.9	-6.7	-5.2	-3.8	-7.8	0.1
V_4	-4.0	0.0	5.2	-8.8	-2.9	5.2	2.6
V_5	-19.0	-4.2	-3.2	-21.1	-5.7	-4.3	1.1
V_6	10.1	4.6	7.3	3.0	2.7	8.2	1.6

Differences between the values of the spatial distance of the side 4-5 (Table 4) are not large – on the order of single millimetres, but they may be significant when examining the relative displacements of the object. The greatest length deviation (2.6 mm) occurs for the V_4 variant (a reference to two points: *WODZ* and *KATO*), which may suggest errors in measuring vectors for the *WODZ* station, because for the V_5 version with a single reference to the *KATO* station this deviation is much smaller (1.1 mm). On the other hand, however, in version V_6 (reference to two points: *WODZ* and *KRAW*) the deviation is not very large (1.6 mm), which does not raise any objections regarding the reference to the *WODZ* station. The recorded deviations are not correlated with the

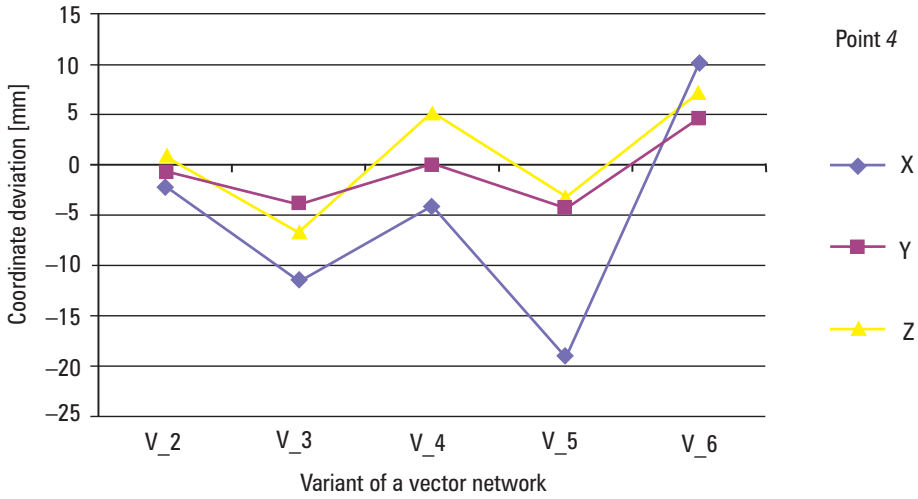


Fig. 2. Coordinate deviations for the point no. 4 (with reference to variant V_1)

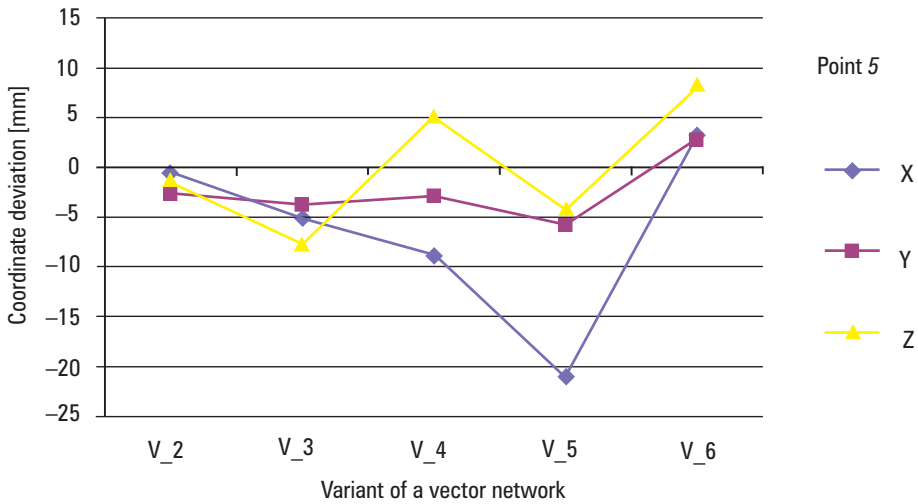


Fig. 3. Coordinate deviations for the point no. 5 (with reference to variant V_1)

number of reference stations used: for versions V_3, V_4 and V_6 (2 stations each), the deviations are differentiated as follows (mm): 0,1; 2,6; 1,6, respectively. On the other hand, when comparing two variants that differ in the directions of the reference (V_3 and V_6), we notice a quite significant discrepancy (0.1 mm – 1.6 mm), which could argue in favour of referring to the N-S directions as a more advantageous option than in the case of the reference in the C-E directions.

When analysing the values of coordinate deviations (Table 4, Fig. 2 and 3), much greater discrepancies than in the case of length deviations can be noticed. The highest values of deviations (in terms of absolute values) were obtained in the variant V_5, especially for the X coordinate of approx. 2 cm – in the case of both points (4 and 5). Errors of this size may significantly distort the results of the absolute displacement test. However, the reasons for such large deviations cannot be found in the errors of a single reference *KATO* station, as it also occurs in all other variants. When we take into account the length deviation for the V_5 variant – 1.1 mm – it also cannot be objected to. Comparing the two variants with different reference directions (V_3 and V_6), it can be noticed that the absolute values of the coordinate deviations are similar, but with opposite signs. For example, the X coordinate for the point no. 4 differs in both variants by more than 2 cm, and for Y – approx. 9 mm. This observation may confirm the theory about the influence of the reference directions (N-S; W-E) on the accuracy of determining the position (similarly to the analysis of length deviations). However, taking into account the number of reference points (V_2 – 3 points; V_4 – 2 points; V_5 – 1 point), a quite clear relationship can be seen: the more reference points, the smaller the deviations. This observation seems quite obvious, but it is not confirmed at the earlier stage of the analysis of deviations for the side length 4-5.

As part of the accuracy assessment after adjustment, the accuracy point parameters were calculated (for points 4 and 5): mean coordinate errors (8) and point position error in space (10). The results of these calculations are presented in Table 5 and illustrated in graphs (Fig. 4 and 5).

Table 5. Accuracy parameters for the determined points (after adjustment)

Version	Mean errors of coordinates [mm]						Position error [mm]	
	Point: 4			Point: 5			Point: 4	Point: 5
	m_x	m_y	m_z	m_x	m_y	m_z	m_{p4}	m_{p5}
V_1	5.0	3.9	4.4	7.5	5.7	6.1	7.7	11.2
V_2	6.5	5.0	5.6	9.9	7.1	7.7	9.9	14.4
V_3	5.9	4.4	4.9	8.0	5.6	6.1	8.9	11.6
V_4	7.6	6.1	6.7	12.5	9.9	10.4	11.8	19.0
V_5	3.3	2.6	2.8	4.8	3.8	4.0	5.1	7.3
V_6	5.2	4.3	4.8	7.2	6.0	6.3	8.3	11.3

Differences in the mean errors of the X, Y, Z coordinates (for individual variants) are small – on the order of 1-2 mm (Table 5). For both, the item no. 4 and the item no. 5, the values of the considered accuracy parameters differ between individual variants, but they are not large. For example, the greatest difference in the position error occurs between variants V_4 and V_5 – approx. 7 mm (point 4) and approx. 12 mm

(point 5). However, such values may be important when assessing the significance of displacements of the tested object. The above comparison leads to the conclusion that the best solution (in terms of accuracy) is given by the variant V_5 (referring to a single point in the N direction), and the worst is the variant V_4 (referring to two points in the directions: W, N) – in this case the position error for item no. 5 is almost 2 cm. The comparative criteria considered do not “fit” so far to such an assessment of these two variants – neither the number of reference points, nor the directions to the reference points (W-E; N-S). This may mean (for example) an inadequate estimation (at the post-processing stage) of the mean errors (a priori) of the measurement of vectors between points of the control network (4, 5) and the KATO station (measurement errors translate into errors of values determined on their basis, i.e. coordinate errors). Also note that the accuracy characteristics (mean errors) for point no. 5 are slightly higher (for all variants) than for point no. 4. This may indicate (for example) the differentiation of errors made when positioning the receivers over these points (centring and antenna height measurement errors).

For the spatial length of the test side 4-5, the length error after adjustment m_d (13) was also calculated (for individual variants), as well as the length deviation d from its reference value d_i (Table 6, Fig. 6). The reference length (d_i) was determined on the basis of accurate (repeated) classical measurements: horizontal distance measurement using a precision total station; measurement of the difference in height using the precision levelling method (electronic level and invar code staffs).

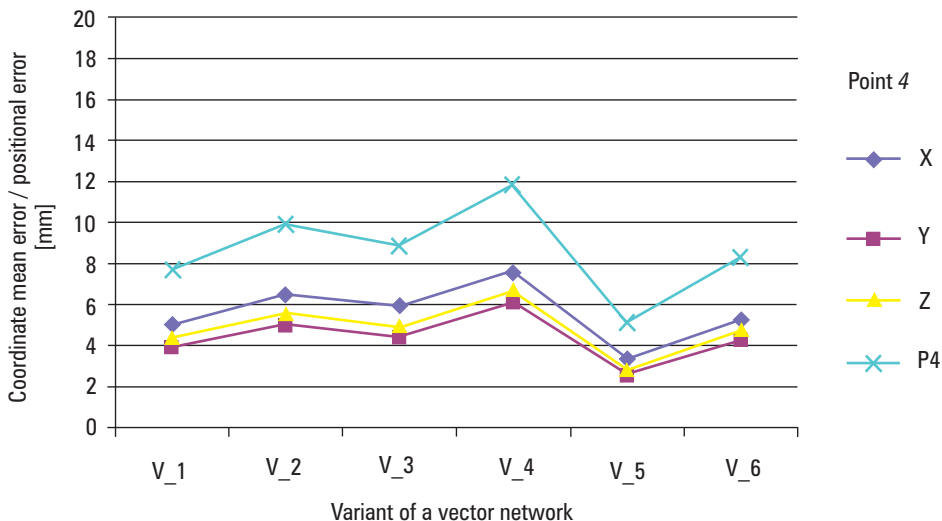


Fig. 4. Point no. 4 location accuracy

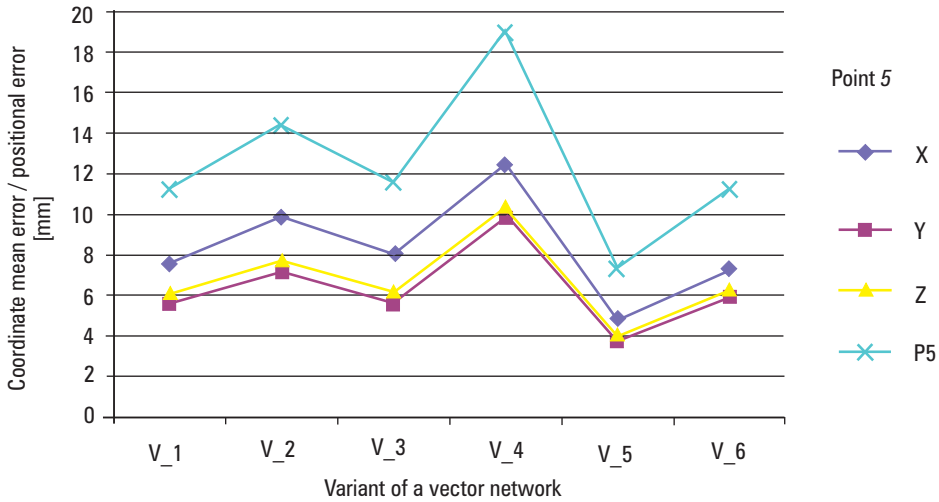


Fig. 5. Point no. 5 location accuracy

Table 6. Analysis of the accuracy of the determined length (d – tested length, d_i – reference length, m_d – length mean error after adjustment)

Version	4–5 side length [m]		Length difference [mm]	Mean error [mm]
	d_i	d	$(d - d_i)$	m_d
V_1	24.4403	24.4303	-10.0	5.9
V_2	24.4403	24.4318	-8.4	7.1
V_3	24.4403	24.4304	-9.9	5.4
V_4	24.4403	24.4329	-7.4	8.8
V_5	24.4403	24.4314	-8.9	3.0
V_6	24.4403	24.4319	-8.3	5.5

The length deviation (determined on the basis of adjusted GNSS vectors) from the reference value (based on classical measurements) is quite large and of a similar value for all variants ($-7 \div -10$ mm) – Table 6. Therefore, assuming that the reference length is error-free, the reasons for these deviations should be found in GNSS measurement errors (e.g. caused by the variability of antenna phase centres – at the points of the control network and at the ASG-EUPOS reference station points). Additionally, the greatest (absolute) deviation is obtained for the variant V_1 (10 mm), where the points of the control network were determined based on the largest number of reference points. Thus, this parameter (the $d - d_i$ deviation) seems useless for the purpose of the accuracy of determining the test length of the side 4–5.

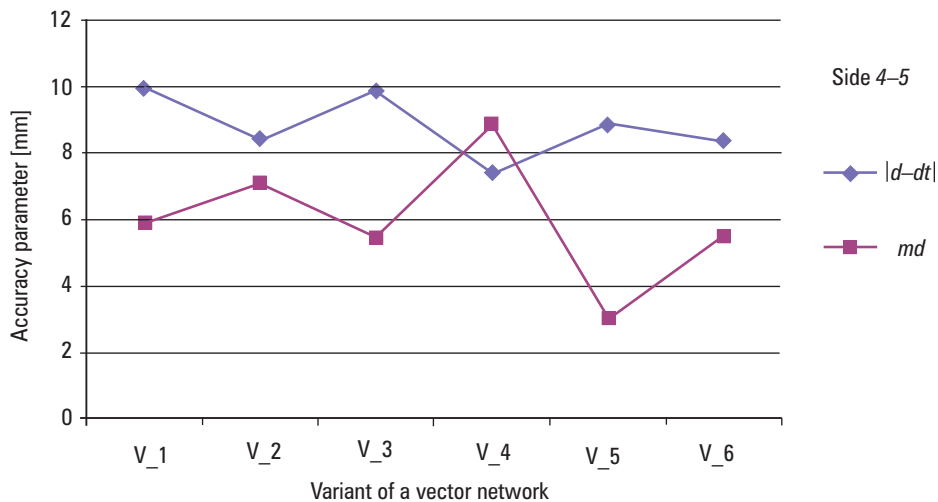


Fig. 6. Accuracy of determining the test length (d – tested length, d_t – reference length, m_d – mean length error after adjustment)

Taking into account the error of a posteriori length m_d – Table 6), the most favourable should be the variant V_5 (reference to a single station KATO), and the worst – variant V_4 (reference to two stations in the directions: W, N), which correlates with the error values of the position of the points 4, 5 (Table 5). Thus, the conclusions here will be consistent with the results of the previous analysis (Table 5, Fig. 4, 5).

4. Conclusions

As a result of independent strict adjustment for each of the six selected test variants, quite significant discrepancies in the coordinates of the designated points were found (up to more than 2 cm), as well as some discrepancies in the distance between these points (max. 2.6 mm). The individual variants also differ from each other at the stage of assessing the accuracy of the compensated quantities – calculation of mean coordinate errors and position errors (discrepancies up to 12 mm) and mean distance errors (discrepancies up to 5 mm). Differences of this order should not be underestimated, as they may be important when determining displacements, especially absolute displacements (coordinates).

The reasons for the discrepancies obtained in the results of adjustment and accuracy assessment can be found not only in the conditions of reference to the reference points, but also in the potential errors that occur during the measurement process of the vector network (e.g. antenna centring and measurement errors; variability of antenna phase centres – especially with different types of antennas at the points of the control network and at the points of ASG-EUPOS reference stations; incorrect estimation of the mean errors of vector measurements, etc.).

On the basis of the performed comparative analyses, no clear relationships were found between the method of establishing the control network to the reference stations (number of stations and their location) and the accuracy of the determinations. For several tested variants, however, the influence of the reference directions (N-S, W-E) on the accuracy of determining the position can be noticed, as well as a quite obvious relationship – the more reference points, the smaller the coordinate deviations. To confirm these conclusions, however, further test studies are needed, carried out for a larger number of (designated) control points and taking into account the detailed conditions of establishing (e.g. control network adjustment based on individual ASG_EUPOS stations – for all possible variants). This research will be continued by the author of this paper.

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