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ANALYSIS OF THE CURVATURE OF THE RIVER BEND FOR REGULATION PURPOSES

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Summary

In order to determine the most appropriate geometrical parameters of regulated watercourses, this paper presents some ways to identify the natural course thereof. In addition to analysing the types of curves approximating the test object, the authors have also described the methods of studying the range of these curves.

In order to determine the degree of relationships between the variables of the studied watercourses, the authors have analysed the correlation coefficients.

The paper presents the findings pertaining to the best-matched curves approximating the given object.

Keywords

approximation • regulation of watercourses

1. Introduction

The need for the modernization of waterways occurs especially when remodelling the geometry of existing routes, whose course was disturbed by the action of natural forces and purposeful human activity.

Issues of design and modernization of waterways were dealt with by many Polish and foreign scholars. Among the especially noteworthy, we should mention R. Grabowski, A. Kobryń, Z. Piasek, J. Czaja in Poland, while international researchers include R. Aüberlen, L. Forque and K. Göldner.

The modernization of water transportation routes is served by identifying the geometric parameters of the route, in order to:

- determine the shape of the individual segments of the route,
- determine the scope of these segments,
- indicate connection points of all kinds approximating curves,
- indicate the possibility of approximating elements of the watercourse on the basis of the data from field measurements or those obtained from the map in the appropriate scales.

An important condition for shaping a given water route is to ensure its smoothness, which is reflected in the selection of a suitable ratio between individual elements of the route.

Among the recommended solutions for curved passages on waterways, the preferable are those that provide the entry into the curve, which is as gentle as possible [Grabowski, Kobryń1987].

From the point of view of the dynamics of movement on the curve, it is necessary that the angle acceleration of tangent rotation at the starting point of the curve should equal zero [Lambor 1960].

Providing these conditions, especially in the case of larger watercourses with concave banks, prevents a situation in which a sudden rise in the water Table would naturally occur, resulting in a significant deformation of the riverbed [Piasek 1995, Piasek 2000].

Studies by the authors who deal with the aforementioned issues indicate, that it is necessary to shape the regulatory route of the river in such a way that would refer to its direction, and to imitate the natural curves of the watercourse [Forque 1908, Göldner 1961]. Thus the importance of selecting the appropriate curve, approximating the natural course of the river.

2. Objectives of the study

The work has two objectives:

1. Overall objective: to analyse the curvature of the banks of the object (watercourse) with the view to regulation
2. Specific objectives:
   a) to determine the shape of the sections of the studied bank of the river,
   b) to identify the range of individual sections.

Among others, the following approximation functions can be used: parabolas of different degrees, spiral curves like the Archimedes spiral, the logarithmic curve and the Cornu spiral, trigonometric curves, exponential curves, and the lemniscate of Bernoulli.

3. Determining the shape of approximating sections

The starting point in solving the problem is a set of coordinates $\{x_i\}$ and $\{y_i\}$ of points representing the studied object. These coordinates are read from the map in the adopted local coordinate system. They can also come from field measurements – classic or satellite.

Regression model should be defined between such variables, for which there is a strong correlation. Measure is Pearson's complete correlation coefficient r [J. Czaja 1996].

The linear correlation coefficient is expressed with the following formula:

$$r = \frac{\mathrm{cov}(X,Y)}{\sigma(Y)\cdot\sigma(X)} \quad \text{[J. Czaja 1996]} \tag{1}$$

while non-linear correlation coefficient is expressed as:

$$r_0 = 1 - \frac{\sum_i [y_i - f(x_i)]^2}{\sum_i [y_i - \bar{y}]^2}$$ [Krysicki, Włodarski i in. 1996; J. Czaja 1993, 1996] (2)

where:

$\sum_i [y_i - f(x_i)]^2$ is the measure of correlation between the nonlinear regression model and empirical values,

$\sum_i [y_i - \bar{y}]^2$ dispersion of variable y_i versus its average value $\bar{y}$.

The closer the absolute value of correlation coefficient is to 1, the stronger the correlation between the studied variables.

Regression equation can be expressed as line regression:

$$y = a_0 + a_1 x \tag{3}$$

or as any curve described by the general formula of:

$$y = f(x) \tag{4}$$

Parameters of these functions are determined using Gauss's method of least squares.

The criterion of least squares makes it possible to find the one line, for which the sum of squares of the deviations of observed values from that line would be the smallest, that is:

$$F = \sum_i [y_i - f(x_i)]^2 = min \tag{5}$$

In order to approximate the curve of the studied section, the models of the following functions were used:

1. Second degree parabola from the family

$$y = A \cdot x^2 + Bx + C \tag{6}$$

2. Logarithmic function

$$y = A + B \cdot ln\, x \tag{7}$$

3. Exponential function

$$y = A \cdot e^{Bx} \tag{8}$$

4. Power function

$$y = A \cdot x^B \tag{9}$$

Ad 1. Approximation of the studied object using the parabolic function

Out of the family of parabolic functions, the parabola was determined which, for the given set of points, minimises the function:

$$F(A,B,C) = \sum_i \left[y_i - (Ax_i^2 + Bx_i + C \right]^2 \tag{10}$$

In order to determine the values of parameters A, B and C of the function (10), partial derivatives of the function were calculated and set equal to zero; subsequently, the resultant system of equations was solved.

From solving the system of equations:

$$\frac{\partial F}{\partial A} = 0 \qquad \frac{\partial F}{\partial B} = 0 \qquad \frac{\partial F}{\partial C} = 0$$

the values of parameters A, B and C were derived. The equations were formulated by [Krupiński W. 1997].

For the set of coordinates $\{x_i\}$ and $\{y_i\}$ of points representing the studied object, the function parameters were determined:

A = 0,0099, B = –3.60, C = 441.26

Thus the approximated parabolic function takes the following format:

$$y = 0{,}0099x^2 - 3.60x + 441.26 \tag{11}$$

Ad 2. Approximation of the studied object using the logarithmic function

Function minimisation:

$$F(A,B) = \sum_i \left[y_i - (A + B \cdot ln x_i) \right]^2$$

$$\frac{\partial F}{\partial A} = 2\sum_i (y_i - A - B \cdot ln x_i) \cdot (-1)$$

$$\frac{\partial F}{\partial B} = 2\sum_i (y_i - A - B \cdot ln x_i) \cdot (-ln x_i) \tag{12}$$

$$\frac{\partial F}{\partial A} = 0 \Leftrightarrow \sum_i A = \sum_i y_i - \sum_i B \cdot ln x_i$$

$$n \cdot A = \sum_i y_i - \sum_i B \cdot ln x_i$$

$$A = \frac{\sum_i y_i - \sum_i B \cdot ln\,x_i}{n}$$

$$\frac{\partial F}{\partial B} = 0 \Leftrightarrow \sum_i B \cdot ln^2\,x_i = \sum_i y_i \cdot ln\,x_i - \sum_i A \cdot ln\,x_i$$

$$B \cdot \sum_i ln^2\,x_i = \sum_i y_i \cdot ln\,x_i - A \cdot \sum_i ln\,x_i \tag{13}$$

$$B = \frac{\sum_i y_i \cdot ln\,x_i - A \cdot \sum_i ln\,x_i}{\sum_i ln^2\,x_i}$$

$$B = \frac{\sum_i y_i \cdot ln\,x_i - \frac{\sum_i y_i - B\sum_i ln\,x_i}{n} \cdot \sum_i ln\,x_i}{\sum_i ln^2\,x_i} \quad |\cdot \sum_i ln^2\,x_i$$

$$B \cdot \sum_i ln^2\,x_i = \sum_i y_i \cdot ln\,x_i - \frac{\sum_i y_i \sum_i ln\,x_i - B\sum_i ln\,x_i \sum_i ln\,x_i}{n} \quad |\cdot n$$

$$n \cdot B \cdot \sum_i ln^2\,x_i = n \cdot \sum_i y_i \cdot ln\,x_i - \sum_i y_i \sum_i ln\,x_i + B\sum_i ln\,x_i \sum_i ln\,x_i$$

$$n \cdot B \cdot \sum_i ln^2\,x_i - B\sum_i ln\,x_i \sum_i ln\,x_i = n \cdot \sum_i y_i \cdot ln\,x_i - \sum_i y_i \sum_i ln\,x_i \tag{14}$$

$$B \cdot \left(n \cdot \sum_i ln^2\,x_i - \sum_i ln\,x_i \sum_i ln\,x_i \right) = n \cdot \sum_i y_i \cdot ln\,x_i - \sum_i y_i \sum_i ln\,x_i$$

$$B = \frac{n \cdot \sum_i y_i \cdot ln\,x_i - \sum_i y_i \sum_i ln\,x_i}{n \cdot \sum_i ln^2\,x_i - \sum_i ln\,x_i \sum_i ln\,x_i}$$

Using formulas (13) and (14), parameters for logarithmic function were calculated for the object:

A = 794.60, B = –131.93

Thus the approximated logarithmic function takes the following format:

$$y = 794.60 = -131.93 \cdot lnx \tag{15}$$

Ad 3. Approximation of the studied object using the exponential function

This can be done using classical method, via function minimisation:

$$F(A, B) = \sum_i \left[y_i - A \cdot e^{Bx_i} \right]^2$$

Therefore, calculated as:

$$\frac{\partial F}{\partial A} = 2\sum_i \left(y_i - A \cdot e^{Bx_i} \right) \cdot e^{Bx_i}$$

$$\frac{\partial F}{\partial A} = 2\sum_i \left(y_i - A \cdot e^{Bx_i} \right) \cdot A \cdot e^{Bx_i} \cdot x_i$$

set as zero, and solve the resultant system of equations.

Also, logarithmic method can be used, proposed by professor J. Czaja [J. Czaja 1996], leading to linear function as follows: by logarithmising both sides of the function:

$$y = \mathrm{A} \cdot e^{\mathrm{B}x}$$

we arrive at:

$$lny = ln\mathrm{A} + \mathrm{B}x \cdot lne = ln\mathrm{A} + \mathrm{B}x$$

and by introducing the symbols:

$$ln\mathrm{A} = \mathrm{C}$$

we arrive at:

$$z = \mathrm{B}x + \mathrm{C}$$

Formulating equations for regression coefficient for the straight line

$$a_0 = \mathrm{C}$$

$$a_1 = \mathrm{B}$$

[Krysicki, Włodarski et al. 1996] we calculate the approximated parameters of the exponential function.

Thus we arrive at the parameters of the exponential function.

$$\mathrm{A} = 359.95,\ \mathrm{B} = -0.0065$$

$$lny = ln\mathrm{A} + \mathrm{B}x$$

and the exponential function approximated for the given object takes the following format:

$$y = 359.95 \cdot e^{-0.0065} \qquad (16)$$

Ad 4. Approximation of the studied object using the power function

Just as for the approximation by exponential function, classical method can be applied, by using partial function derivatives:

$$F(A, B) = \sum_i \left[y_i - A \cdot x_i^B \right]^2$$

where:

$$\frac{\partial F}{\partial A} = 2\sum_i \left(y_i - A \cdot x_i^B\right) \cdot x_i^B$$

$$\frac{\partial F}{\partial B} = 2\sum_i \left(y_i - A \cdot x_i^B\right) \cdot A \cdot x_i^B \cdot \ln x_i$$

and further calculations can be made, as in the previous case.

The logarithmic method can also be used [Czaja 1996] as shown below:
by logarithmising the function:

$$y = \mathrm{A} \cdot x^{\mathrm{B}}$$

$$\ln y = \ln \mathrm{A} + \ln \mathrm{x}$$

and introducing the symbols:

$$\ln y = z$$

$$\ln \mathrm{A} = \mathrm{C}$$

$$\ln x = t$$

we arrive at:

$$z = \mathrm{B}t + \mathrm{C}$$

The estimated parameters of the power function are calculated in the same way as in the case of the exponential function.

Thus we obtain the parameters of the object:

A = 5560.55, B = –0.7519

while the approximated power function takes the following format:

$$y = 5560.55 \cdot x^{-0.7519} \tag{17}$$

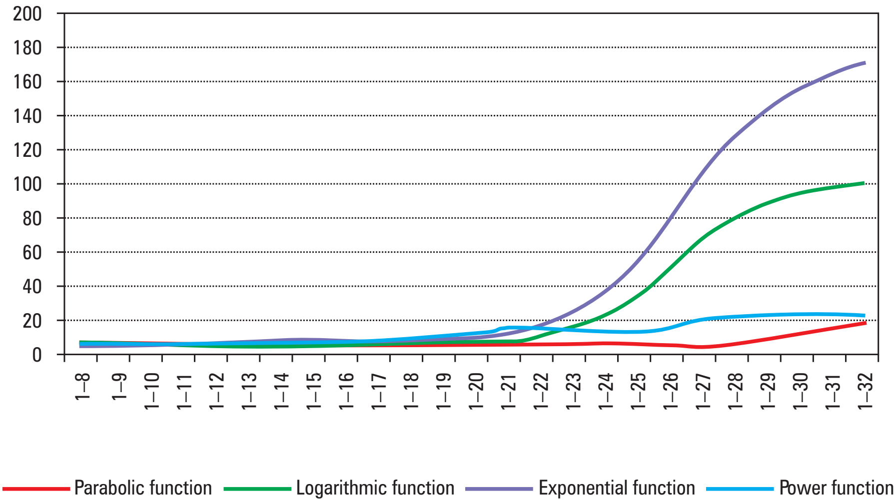
Reading from Table 1 the values of $\frac{[y - f_i(x_i)]^2}{n}$ for all functions approximating the studied object, we note that the best approximating function is the parabolic function – having the value of 19.11; the next would be the power function – with the value of 23.45; while the logarithmic and exponential functions would not provide a good approximation of the natural course of the studied object.

Table 1. List of the parameters of the analysed functions and their correlation coefficients.

Function	Range of points used for determining the parameters	A	B	C	r_0	$\frac{[y - f_i(x_i)]^2}{n}$
Parabolic function	1–32	0.0099	–3.60	441.26	0.9972	19.11
Logarithmic function	1–32	794.60	–131.93	–	0.9852	100.53
Exponential function	1–32	359.95	–0.0065	–	0.9467	171.58
Power function	1–32	5660.55	–0.7519	–	0.9467	23.45

Using the above calculation methods, the sums of square deviations from the functions were analysed, for increasingly smaller fragments (sections) of the object. These values are plotted in Figure 1.

Our conclusions as to the best fit of the parabolic function is also confirmed by Figure 1, showing the graphs $\frac{[y - f_i(x_i)]^2}{n}$, which have been determined for the parts parts of the studied section.



Source: authors' study

Fig. 1. Graphs $\frac{[y - f_i(x_i)]^2}{n}$ of the studied functions

Thus we obtained the answer to our first question, that is, the determination of the shape of the studied bank of the river.

4. Identifying the range of the approximating function

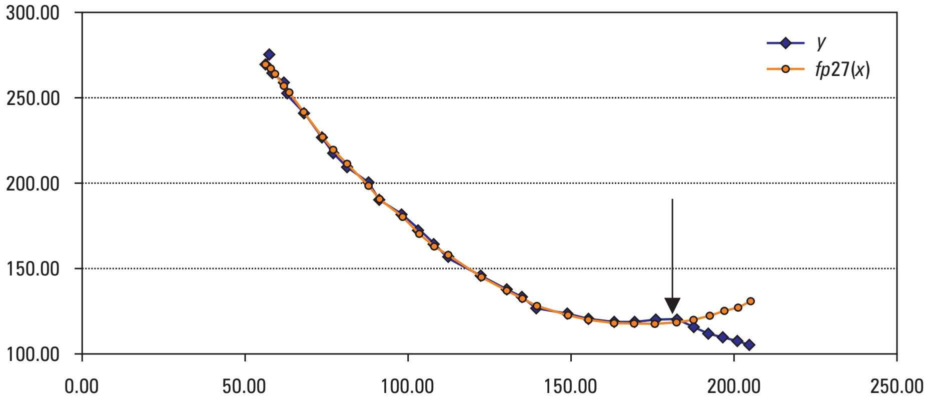
At this point we shall focus on the parabolic function, which – according to the investigation presented above – best approximates our object.

The examination of the range of the curves in question is illustrated in Figure 2 and Table 2.

Points 1–28 belong to the approximated function $f_{p28}(x)$. When values of $[y - f_{p28}(x)]$ are starting to increase 'significantly,' then we can assume that from this point the studied object would be better approximated by another function, for example $f_2(x)$. This will happen at point P.

Table 2. Examination of the range of the parabolic curve

No.	x	y	$fp28(x)$
1	57.11	276.56	269.57
2	56.21	270.73	271.97
3	58.15	264.94	266.82
4	62.13	259.41	256.48
5	62.91	254.42	254.49
6	62.56	253.89	255.38
7	62.21	255.42	256.28
8	67.75	241.89	242.53
9	73.70	227.83	228.58
10	76.86	217.92	221.49
11	81.06	209.23	212.44
12	87.45	201.57	199.48
13	90.98	190.00	192.74
14	97.60	182.87	180.88
15	102.86	174.02	172.18
16	107.62	165.71	164.87
17	111.66	157.09	159.08
18	122.02	147.03	146.00
19	129.60	139.59	138.04
20	134.57	134.21	133.55
21	139.13	126.62	129.93
22	148.71	124.83	123.93
23	154.93	121.25	121.19
24	162.32	120.12	119.12
25	168.74	118.88	118.36
26	175.29	120.84	118.59
27	181.63	120.81	119.77
28	186.71	117.45	121.39
29	191.45	112.67	123.46
30	196.09	110.97	125.99
31	200.32	108.40	128.74
32	204.53	105.48	131.89



Source: authors' study

Fig. 2. Examination of the range of the parabolic curve

In our particular example, on the basis of Table 1, we can conclude that for the approximated parabolic function:

$$y = 0.0099x^2 - 3.60x + 441.26$$

this will be point 27 or 28.

In order to verify the result, and achieve a more accurate determination of the point that we are looking for, we have shown, in Table 2, value difference and quotients of the values of $\frac{[y - f_i(x_i)]^2}{n}$.

The values were collated starting from point 20, according to the data presented in Figure 1.

Table 3. The list of value differences $\frac{[y - f_i(x_i)]^2}{n}$ and value quotients $\frac{[y - f_i(x_i)]^2}{n}$

Value differences $\frac{[y - f_i(x_i)]^2}{n}$		**Value quotients** $\frac{[y - f_i(x_i)]^2}{n}$	
21–20	0.62	20 : 21	0.90
22–21	–0.26	21 : 22	1.05
23–22	–0.24	22 : 23	1.04
24–23	–0.23	23 : 24	1.04
25–24	–0.20	24 : 25	1.04
26–25	–0.17	25 : 26	1.03
27–26	–0.17	26 : 27	1.03
28–27	0.65	27 : 28	0.88
29–28	2.65	28 : 29	0.67
30–39	2.89	29 : 30	0.74
31–30	3.62	30 : 31	0.75
32–31	4.53	31 : 32	0.76

Values contained in Table 3 confirm the previous conclusion that the point 27 or 28 still belongs to the determined parabolic function.

In order to determine more precisely the function approximating the further course of the studied object, one would need to look for another function.

5. Conclusions

General:

1. Activities related to the proper modernization of waterways should begin with the determination of the natural courses.
2. In order to determine the natural course of the waterways, approximation tests should be conducted on various types of curves, from the families of exponential, logarithmic, parabolic, spiral and other curves.
3. Approximation should be carried out on the basis of the coordinates of the shoreline points, derived from site measurements or read from the map in the appropriate scales,

Specific:

4. Studies conducted at the site – on the object of river Wilga – showed that among the parabolic, logarithmic, exponential, and power functions – it is the parabolic function that provides the best approximation; of the following format:

$$y = 0.0099x^2 - 3.60x + 441.26$$

5. Another approximation function can be the exponential function described by the formula:

$$y = 5560.55 \cdot x^{-0.7519}$$

6. Furthermore, the investigated functions from the family of exponential and logarithmic functions would not provide a good approximation of the natural course of the studied object.
7. The method for examining the range of the parabolic function, as the one best defining the object shown in Figure 2 and Table 3, shows that points 27 and 28 still belong to the studied parabola, while another approximation would be necessary to determine the curve to which the next points belong.

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