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APPLICATION OF SELECTED STATISTICAL TESTS IN LEVELLING STUDIES

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Summary

The following paper presents a comparative application of three selected tests (L'Abbé's, Kolmogorov's, and Kuiper's) in order to verify the normality of the distribution of measurement errors in precise levelling. The calculations were made on the basis of observations obtained from the levelling test network, established at the LZD experimental station in Krynica.
The tests subjected to the comparison have demonstrated that the distribution of error in the studied tests is a normal distribution, and that there are no systematic errors in the measurements.

Keywords

mathematical statistics • statistical tests • analysis of measurement accuracy • precise levelling

1. Introduction

On the test grid established at the Forest Experimental Station (LZD) in Krynica, high-precision levelling measurements were made in a test study, in order to answer the question regarding the choice of the method for network adjustment.

As is well known, when surveying observations follow a normal distribution of measurement errors, then the Gaussian / least squares' method can be used for their alignment [Puchalski 1969].

Otherwise, if the geodetic observations were subject to systematic errors, that is if the distribution of measurement errors were not in line with the theoretical normal distribution, then in order for the exact alignment of the network to be achieved, it would be necessary to compensate and apply a different equation procedure than the Gaussian method, for instance, the method of maximum likelihood estimation [Kasietczuk 1993, Kamiński 2000].

Therefore, the main goal of the present study was to check whether the error distributions of altitude measurements made within the test network are consistent with the theoretical normal distribution.

2. Description of the experiment

The test grid consists of seven levelling lines, forming three closed “circuits”.

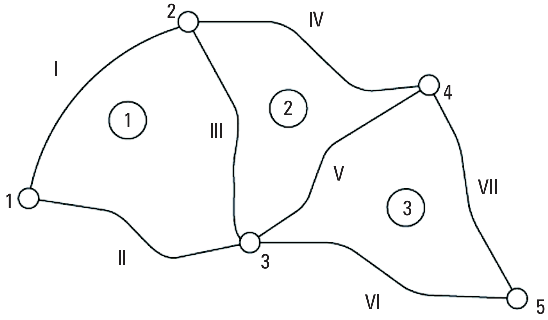

Fig. 1. Test grid for precise levelling

In order to solve the posited problem, the height measurements of each closed levelling line were repeated four times, using two precise Zeiss levels: Ni 002 No. 219422 and Ni 007 No. 547355.

This resulted in a statistical sample of 12 closure errors of levelling circuits, for each instrument.

For statistical tests, tests such as: λ – Kolmogorov’s, χ^2 – Pearson’s, or Student’s *t*-distribution are typically applied [Ney B. 1970] , [Greń J. 1970], [Siejka M. 2017].

In this work, other tests will be presented, which are less frequently used in practice, namely:

1) L’Abbé’s statistical hypothesis test [Godek K., Krupiński W. 2010],
2) Kolmogorov’s *D* and *D’* test [Kasietczuk B. 1993],
3) Kuiper’s V and V_1 test [Kasietczuk B. 1993].

3. L’Abbé’s test

Due to the fact that the theoretical basis of L’Abbé’s test has been provided in the publication [Godek and Krupiński 2010], here we shall confine ourselves to providing the method of calculation. By means of the aforementioned test, it is possible to verify whether the distribution of error in the surveying measurements, which we put to the test, is consistent with normal distribution, and whether it is not burdened with systematic errors that would distort these measurements (5).

We propose to test the null hypothesis – being:

$\mathbf{H}_0$: the distribution of errors corresponds to normal distribution

– against the alternative hypothesis, namely:

$\mathbf{H}_1$: the tested distribution is not a normal distribution,

In order to do that, we need perform the following tasks:

1. Calculate the m_d^2 characteristics, using the following formula:

$$m_d^2 = \frac{1}{2(n-1)} \cdot \sum_{i=1}^{n-1} d_i^2 \text{ where } d_i = x_{i+1} - x_i \tag{1}$$

2. Compute the standard deviation m^2, using the following formula:

$$m^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} v_i^2 \text{ where } v_i = x_i - \overline{x} \tag{2}$$

3. Calculate the quotient:

$$\frac{m_d^2}{m^2} = \tau \tag{3}$$

4. Read the value from L'Abbé's Plot:

$$\tau_{\alpha/n} = \tau_q$$

If $\tau > \tau_q$ – then the examined distribution corresponds to the normal distribution and there are no grounds for rejecting H_0.

If $\tau < \tau_q$ – H_0 in favour of H_1.

The calculations for the L'Abbé's test are given in Tables 1 and 2.

If $\tau < \tau_q$ – then the examined distribution does not correspond to the normal distribution, then we reject H_0 in favour of H_1.

Computations for L'Abbé's test are given in Tables 1 and 2.

Table 1. Computations for L'Abbé's test for level Ni 002

i	x_i	d_i	d_i^2	v_i	v_i^2
	[mm]				
1	–2.9	1.6	2.56	–2.8	7.84
2	–1.3	2.1	4.41	–1.2	1.44
3	0.8	0.9	0.81	0.9	0.81
4	1.7	–1.4	1.96	1.8	3.24
5	0.3	–0.8	0.64	0.4	0.16
6	–0.5	2.0	4.00	–0.4	0.16
7	1.5	–3.6	12.96	1.6	2.56
8	–2.1	1.5	2.25	–2.0	4.00
9	–0.6	2.0	4.00	–0.5	0.25

Table 1. cont.

i	x_i	d_i	d_i^2	v_i	v_i^2
10	1.4	0.8	0.64	1.5	2.25
11	2.2	–3.6	12.96	2.3	5.29
12	–1.4			–1.3	1.69
Σ	–0.9		47.19	0.3	29.69

$\bar{x} = -0.1$
$\Sigma d_i^2 = 47.19 \quad \Sigma v_i^2 = 29{,}69$
$m_d^2 = 2.15 \quad m^2 = 2.79$
$\tau = 0.7963; \quad \tau_q = \tau_{0.05/12} = 0.7461$

Table 2. Computations for L'Abbé's test for level Ni 007

i	x_i	d_i	d_i^2	v_i	v_i^2
	[mm]				
1	–3.2	5.8	33.64	–2.9	8.41
2	2.6	–1.3	1.69	2.9	8.41
3	1.3	–0.7	0.49	1.6	2.56
4	0.4	–3.1	9.61	0.7	0.49
5	–2.7	2.8	7.84	–2.4	5.76
6	0.1	–1.1	1.21	0.4	0.16
7	–1.7	0.9	0.81	–0.9	0.81
8	–2.1	5.2	17.04	–1.8	3.24
9	3.1	–1.8	3.24	3.4	11.56
10	1.3	–3.8	14.44	1.6	2.56
11	–2.5	1.5	2.25	–2.2	4.84
12	–1.0			–0.7	0.49
Σ	–3.9		102.26	–0.3	49.29

$\bar{x} = -0.3$
$\Sigma d_i^2 = 102.26 \quad \Sigma v_i^2 = 49{,}29$
$m_d^2 = 4.65 \quad m^2 = 4.48$
$\tau = 1.0268 \quad \tau_q = \tau_{0.05/12} = 0.7461$

As follows from the above:
for Ni 002:

$$\tau = 0.7963 > 0.7461 = \tau_q \text{ for } \alpha = 0.05 \; n = 12$$

and for Ni 007:

$$\tau = 1.0268 > 0.7461 = \tau_q \text{ for } \alpha = 0.05 \; n = 12.$$

Since L'Abbé's test is a left tailed test, these inequalities testify to the compatibility of empirical distributions for both levels with the theoretical normal distribution, in other words, they indicate the lack of systematic errors in the measurements of the test network.

The order of test calculations for Ni 002 is shown in Table 3, and for Ni 007, in Table 4, respectively.

Formulas for Kolmogorov's D and D' tests, as well as Kuiper's V and V_1 are the following:

From Tables 3 and 4, we read the values of D^+, and D^-, and we compute:

$$D' = \left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}} \right) \cdot D \qquad \text{whereas } D = \max(D^+; D^-)$$

$$V_1 = \left(\sqrt{n} + 0.05 + \frac{0.82}{\sqrt{n}} \right) \cdot V \qquad \text{whereas } V = D^+ + D^-$$

After calculating S and σ (variance and standard deviation), we set the values of $z_i = F(u_i)$, where $F(u)$ – the distribution function of the standard normal distribution, and $u_i = \frac{x_i - \bar{x}}{S}$.

Further calculations are presented in Tables 3 and 4.

4. Kolmogorov's test

Table 3. Computations for Kolmogorov's test, for the observation from level Ni 002

i	x_i	$v_i = x_i - \bar{x}$	v_i^2	$u_i = \frac{v_i}{S}$	$z_i = F(u_i)$	i/n	$(i-1)/n$	$i/n - z_i$ D^+	$z_i - (i-1)/n$ D^-
1	−2.9	−2.8	7.84	−1.04	0.0336	0.0833	0.0000	0.0497	0.0336
2	−2.1	−2.0	4.00	−0.74	0.0901	0.0875	0.0833	−0.0260	0.0068
3	1.4	−1.3	1.69	−0.48	0.1814	0.2500	0.0875	0.0686	0.0939
4	−1.3	−1.2	1.44	−0.44	0.1977	0.3333	0.2500	0.1355	0.0523
5	−0.6	−0.5	0.25	−0.19	0.3336	0.4167	0.3333	0.0831	0.0003
6	−0.5	−0.4	0.16	−0.15	0.3557	0.5000	0.4167	0.1443	−0.0610

Table 3. cont.

i	x_i	$v_i = x_i - \bar{x}$	v_i^2	$u_i = \frac{v_i}{S}$	$z_i = F(u_i)$	i/n	$(i-1)/n$	$i/n - z_i$ D^+	$z_i - (i-1)/n$ D^-
7	0.3	0.4	0.16	0.15	0.5948	0.5833	0.5000	−0.0115	0.0948
8	0.8	0.9	0.81	0.33	0.7088	0.6667	0.5833	−0.0421	0.1255
9	1.4	1.5	2.25	0.56	0.8186	0.7500	0.6667	−0.0686	0.1519
10	1.5	1.6	2.56	0.59	0.8365	0.8333	0.7500	−0.0833	0.0865
11	1.7	1.8	3.24	0.67	0.8643	0.9167	0.8333	0.0524	0.0310
12	2.2	2.3	5.29	0.85	0.9251	1.0000	0.9167	0.0749	0.0084
Σ	−0.9								

$$V = S = \frac{\sum v_i^2}{n-1} = \frac{29.69}{11} = 2.70 \quad D' = \left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}} \right) \cdot D =$$

$= (3.46 - 0.01 + 0.25) \cdot 0.1519 = 0.5620$

$\delta(x) = \sqrt{n} = 1.64; \ \bar{x} = -0.1$

$D^+ = 0.1356; D^- = 0.1519$

$D = \max(D^+; D^-) = 0.1519$

For $\alpha = 0.05$ $D'_{kryt} = 0.896$

Table 4. Computations for Kolmogorov's test, for level Ni 007

i	x_i	$v_i = x_i - \bar{x}$	v_i^2	$u_i = \frac{v_i}{S}$	$z_i = F(u_i)$	i/n	$(i-1)/n$	$i/n - z_i$ D^+	$z_i - (i-1)/n$ D^-
1	−3.2	−2.9	8.41	−0.65	0.0495	0.0833	0.0000	0.0338	0.0495
2	−2.7	−2.4	5.76	−0.53	0.0778	0.0875	0.0833	0.0097	−0.0055
3	−2.5	−2.2	4.84	−0.49	0.0934	0.2500	0.0875	0.1566	0.0059
4	−2.1	−1.8	3.24	−0.40	0.1292	0.3333	0.2500	0.2041	−0.1208
5	−1.2	−0.9	0.81	−0.20	0.2389	0.4167	0.3333	0.1778	−0.0944
6	−1.0	−0.7	0.49	−0.16	0.2709	0.5000	0.4167	0.2291	−0.1458
7	0.1	0.4	0.16	0.09	0.5753	0.5833	0.5000	0.0080	0.0753
8	0.4	0.7	0.49	0.16	0.6293	0.6667	0.5833	0.0374	0.0460
9	1.3	1.6	2.56	0.36	0.7734	0.7500	0.6667	0.0234	0.1067
10	1.3	1.6	2.56	0.36	0.7734	0.8333	0.7500	0.0599	0.0234
11	2.6	2.9	8.41	0.65	0.9147	0.9167	0.8333	0.0020	0.0814
12	3.1	3.4	11.56	0.76	0.9452	1.000	0.9167	0.0548	0.0285
Σ	−3.9	−0.3	49.29	−0.05		6.4208	5.4208		

5. Calculations for Kolmogorov's D+ and D– test and for Kuiper's V and V1 test

$$V = S = \frac{\sum v_i^2}{n-1} = \frac{49.29}{11} = 4.48 \quad D' = \left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}}\right) \cdot D$$

$$= (3.46 - 0.01 + 0.25) \cdot 0.229 = 0.8477$$

$$\delta(x) = \sqrt{v} = 2.12; \ \overline{x} = -0.3$$

$D^+ = 0.2291$; $D^- = 0.1067$ for $\alpha = 0.05$ $D'_{kryt} = 0.896$
$D = \max(D^+, D^-) = 0.2291$

Computation results for Kuiper's V and V_1 test:

$$V = D^+ + D^- \qquad V_1 = \left(\sqrt{n} + 0.05 + \frac{0.82}{\sqrt{n}}\right) \cdot V$$

$$V = 0.1356 + 0.1519 = 0.2875$$

$$V_1 = \left(\sqrt{12} + 0.05 + \frac{0.82}{\sqrt{12}}\right) \cdot V = (3.46 + 0.05 + 0.24) \cdot 0.2875 = 1.0781$$

For $\alpha = 0.05$ and $n = 12$ $D'_{kryt} = 1.489$

$$1.489 > 1.078 \Rightarrow \text{normal distribution}$$

Following from Kolmogorov's D and D' test:

For Ni 002: $D'_{kryt} = 0.896$

$D' = 0.562 \Rightarrow D'_{kryt} > D'$

For Ni 007: $D'_{kryt} = 0.896$

$D' = 0.848 \Rightarrow D'_{kryt} > D'$

Which testifies to the lack of systematic error in the measurement process.

Following from Kuiper's V and V_1 test:

For Ni 002: $V_{1kryt} = 1.489$

$V_1 = 1{,}071 \Rightarrow V_{1kryt} > V_1$

For Ni 007: $V_{1kryt} = 0.896$

$V_1 = 0.848 \Rightarrow V_{1kryt} > V_1$

6. Conclusions

1. From L'Abbé's test:
   for Ni 002: $\tau = 0.7963 > \tau_q = \tau_{0.05/12} = 0.7461$
   for Ni 007: $\tau = 1.0268 > \tau_q = \tau_{0.05/12} = 0.7461$
   and, because this is a left tailed test, it follows that the measurements between the two levels show normal distribution of errors.
2. From Kolmogorov's D and D' test
   for Ni 002: $D' = 0.5620 < 0.896 = D'_{\text{kryt}}$
   for Ni 007: $D' = 0.8477 < 0.896 = D'_{\text{kryt}}$
   which testifies to lack of systematic errors in test measurements.
3. From Kuiper's V and V_1 test:
   for Ni 002: $V_1 = 1.0781 < 1.489 = V_{1\text{kryt}}$
   for Ni 007: $V_1 = 1.2593 < 1.489 = V_{1\text{kryt}}$
   which reaffirms the results of the two previous tests.
4. Since all three tests produced the answer that the error distributions for both levels correspond to normal distribution, it can be concluded that the studied levelling network can be compensated by Gauss's method of least squares.

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